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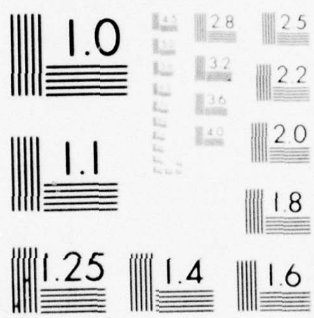

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THE NUMERICAL SOLUTION OF THE DYNAMIC RESPONSE OF HELICAL SPRINGS

SUNIL K. SINHA
AND
GEORGE A. COSTELLO

DECEMBER 1976

U.S. ARMY ARMAMENT COMMAND

RESEARCH DIRECTORATE

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GENERAL THOMAS J. RODMAN LABORATORY
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14

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RIA-R-TR-76-045	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 The Numerical Solution of the Dynamic Response of Helical Springs.		5. TYPE OF REPORT & PERIOD COVERED Final (Nov 75 - May 76)
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Sunil K./Sinha George A./Costello		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Theoretical & Applied Mechanics University of Illinois Urbana, Illinois 61801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMS 3110.15.1114
11. CONTROLLING OFFICE NAME AND ADDRESS Research Directorate, SARRI-RLR-W GEN Thomas J. Rodman Laboratory Rock Island Arsenal, Rock Island, IL 61201		12. REPORT DATE 11 Dec[redacted]976
		13. NUMBER OF PAGES 38
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 9 Final rept. Nov 75 - May 76		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

12 44p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Helical Springs
Impact Springs
Dynamic Response
Analysis
Numerical Solutions

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An analysis was conducted during the period of November 1975 to May 1976 to determine the dynamic response of helical springs under axial impact with particular application to the return spring in the M60A2 tank recoil mechanism. The study was performed by the University of Illinois at Urbana, Illinois under the direction of the General Thomas J. Rodman Laboratory, Rock Island Arsenal. The dynamic behavior of the impacted spring is described by two coupled nonlinear partial differential equations. Two computer solutions to the equations are obtained by the methods of finite differences and nonlinear

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characteristics. Results of the two numerical techniques are compared with the closed form solution of the linear equations. The study indicates that frictional binding may occur between the spring coil diameter and the guide components which will result in failure to return to battery.

Foreword

This report was prepared by George A. Costello[1] and Sunil K. Sinha[2] of the University of Illinois at Urbana, Illinois.

The investigation was a theoretical analysis of the dynamics of helical springs with broad application to all such springs. The investigation was part of a general study to solve the failure to return to battery (FRB) problem associated with the M60A2 tank recoil mechanism. The study was authorized and funded by the Project Manager Office, U.S. Army Tank Automotive Command. The work was conducted under the joint direction of the Research Directorate (Mr. H. Swieskowski) and the Artillery and Armored Weapons Systems Directorate (Mr. L. Neff), General Thomas J. Rodman Laboratory, Rock Island Arsenal, Rock Island, Illinois.

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Contents

INTRODUCTION

In this report an attempt has been made to obtain a numerical solution of two coupled non-linear partial differential equations. Although, the present problem essentially deals with the dynamic response of helical springs, the numerical scheme presented here can be easily extended for the general case of coupled wave equations.

For several coupled one-dimensional waves associated with the displacements $u_1(x,t)$, $u_2(x,t)\cdots u_n(x,t)$, the general system of n non-linear partial differential equations can be written as:

$$
\begin{aligned}
&a_{11}\frac{\partial^2 u_1}{\partial x^2} + \cdots a_{1n}\frac{\partial^2 u_n}{\partial x^2} = \frac{\partial^2 u_1}{\partial t^2} \\
&\cdots\cdots \\
&a_{n1}\frac{\partial^2 u_1}{\partial x^2} + \cdots a_{nn}\frac{\partial^2 u_n}{\partial x^2} = \frac{\partial^2 u_n}{\partial t^2}
\end{aligned} \tag{1}
$$

In the present case, the solution of only two coupled wave equations will be discussed; a situation which arises in the impact of helical springs. However the method can be easily extended to any number of coupled one-dimensional waves.

If one end of a massless spring is displaced relative to the other end and the relative displacement is a function of the time t, then the stresses in the spring are uniform and depend only on the time. The assumption of a massless spring is generally on the non-conservative side as for as stresses are concerned since the stresses are uniformly distributed along the spring. The problem becomes much more complex however if the mass of the spring is taken into account and large displacements are possible.

The static response of helical springs subjected to large deflections is presented in the work of Love [1]. The dynamic response of springs is treated in articles by Johnson [2], Krebs and Weidlich [3], Dick [4], Gaballe [5], Durant [6], Wittrick [7], Britton and Langley [8], Johnson and Stewart [9], Kagawa[10], Suh[11], Pujara and Kagawa[12], Haines and Huang[13], Haines, Chang and Huang[14]. Wth the exception of Love, the above authors restricted their analysis to small displacements about an equilibrium position. Stokes[15], in a recent paper conducted an analytical and experimental program to investigate the radial expansion of helical springs due to longitudinal impact. In an article by Phillips and Costello[16], an experimental and theoretical investigation was made of the large deflections of an impacted spring.

Recent inquiries into the significance of torsional oscillations on the radial expansion of helical springs, prompted the work of Costello[17]. In this work, a linear theory was presented and the solution did indicate rather large radial expansion under impact. It is the purpose of this present report to investigate the non-linear behavior of impacted springs and to compare the results with the linear theory.

Since, the governing equations derived by Phillips and Costello [16] are highly non-linear in nature, the solution of the system of equations can be obtained only by some approximate numerical technique. In this report two numerical techniques are described: (1) the method of non-linear characteristics (2) and the method of finite differences. A comparison is then made with the linear theory.

THEORY

The theory behind the method of characteristics has been discussed in detail by Abbott [18] and it has been used for solving various wave propagation problems by several authors e.q. Chou and Mortimer[19] and Pernica and McNiven[20]. The method of characteristics is capable of handling any time-dependent input for both linear and non-linear wave propagation problems, but most of the reported work has been limited to the method of linear characteristics only. In this present work, consideration is given to the problem of the helical spring in complete non-linear form.

The finite-difference method for solving partial differential equations, is quite classical in nature. The work of Courant, Freidricks and Lewy[21], Freidricks[22], Lees[23], Forsythe and Wasow[24], Lax and Wendroff[25], Collatz[26], Gourly and Morris [27] etc., are excellent references for finite-difference methods for non-linear hyperbolic equations. A recent paper by Smith [28] deals with the solution of the 3-dimensional wave equation by finite differences.

METHOD OF CHARACTERISTICS

Consider a system of two coupled non-linear partial differential equations of the following form:

$$\begin{aligned} a_{11}(u_x, v_x)\frac{\partial^2 u}{\partial x^2} + a_{12}(u_x, v_x)\frac{\partial^2 v}{\partial x^2} &= \frac{\partial^2 u}{\partial t^2} \\ a_{21}(u_x, v_x)\frac{\partial^2 u}{\partial x^2} + a_{22}(u_x, v_x)\frac{\partial^2 v}{\partial x^2} &= \frac{\partial^2 v}{\partial t^2} \end{aligned} \tag{2}$$

with the boundary conditions

$$u_t(0,t) = \phi_1(t) \tag{3}$$

$$u_t(1,t) = \phi_2(t) \tag{4}$$

$$v_t(0,t) = \psi_1(t) \tag{5}$$

$$v_t(1,t) = \psi_2(t) \tag{6}$$

and the initial conditions

$$u_x(x,0) = f_1(x) \tag{7}$$

$$v_x(x,0) = f_2(x) \tag{8}$$

$$u_t(x,0) = 0 \tag{9}$$

$$v_t(x,0) = 0 \tag{10}$$

The above set of equations (2) can be converted into a set of first order partial differential equations

$$a_{11}\frac{\partial u_x}{\partial x} + a_{12}\frac{\partial v_x}{\partial x} - \frac{\partial u_t}{\partial t} = 0 \tag{11}$$

$$a_{21}\frac{\partial u_x}{\partial x} + a_{22}\frac{\partial v_x}{\partial x} - \frac{\partial v_t}{\partial t} = 0 \tag{12}$$

$$\frac{\partial u_x}{\partial t} - \frac{\partial u_t}{\partial x} = 0 \tag{13}$$

$$\frac{\partial v_x}{\partial t} - \frac{\partial v_t}{\partial x} = 0 \tag{14}$$

Also

$$\frac{\partial u_x}{\partial x}dx + \frac{\partial u_x}{\partial t}dt = du_x \tag{15}$$

$$\frac{\partial v_x}{\partial x}dx + \frac{\partial v_x}{\partial t}dt = dv_x \tag{16}$$

$$\frac{\partial u_t}{\partial x}dx + \frac{\partial u_t}{\partial t}dt = du_t \tag{17}$$

$$\frac{\partial v_t}{\partial x}dx + \frac{\partial v_t}{\partial t}dt = dv_t \tag{18}$$

Equations (11)....(18) can be written as follows:

$$
\begin{vmatrix}
a_{11} & 0 & a_{12} & 0 & 0 & -1 & 0 & 0 \\
a_{21} & 0 & a_{22} & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
dx & dt & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & dx & dt & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & dx & dt & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & dx & dt
\end{vmatrix}
\begin{Vmatrix}
\frac{\partial u_x}{\partial x} \\
\frac{\partial u_x}{\partial t} \\
\frac{\partial v_x}{\partial x} \\
\frac{\partial v_x}{\partial t} \\
\frac{\partial u_t}{\partial x} \\
\frac{\partial u_t}{\partial t} \\
\frac{\partial v_t}{\partial x} \\
\frac{\partial v_t}{\partial t}
\end{Vmatrix}
=
\begin{Vmatrix}
0 \\ 0 \\ 0 \\ 0 \\ du_x \\ dv_x \\ du_t \\ dv_t
\end{Vmatrix}
\tag{19}
$$

The characteristic directions are determined by setting the determinate of the coefficient matrix of Eq. 19 equal to zero. Hence, the following equation results

$$
(a_{11} a_{22} - a_{12} a_{21}) \left(\frac{dt}{dx}\right)^4 - (a_{11} + a_{22}) \left(\frac{dt}{dx}\right)^2 + 1 = 0 \tag{20}
$$

The above equation has four roots which are

$$
\left.\frac{dt}{dx}\right)_{1,2} = + \left[\frac{(a_{11} + a_{22}) \pm \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}}{2(a_{11}a_{22} - a_{12}a_{21})}\right]^{\frac{1}{2}} \tag{21}
$$

and

$$
\left.\frac{dt}{dx}\right)_{3,4} = - \left[\frac{(a_{11} + a_{22}) \pm \sqrt{(a_{11} - a_{22})^2 + 4a_{12} a_{21}}}{2(a_{11} a_{22} - a_{12} a_{22})}\right]^{\frac{1}{2}} \tag{22}
$$

Since the system of equations (2) has been assumed to be hyperbolic, all the four roots are real and thus there are four characteristics. In order to determine the differential equations along these characteristics, the first column of the coefficient matrix is replaced by the column on the right hand side of Eq. 19 and the coresponding determinant is set equal to zero. Hence, the following equation results

$$du_x\left[1 - a_{22}\left(\frac{dt}{dx}\right)^2\right] + a_{12}\, dv_x\left(\frac{dt}{dx}\right)^2$$

$$- du_t\left[1 - a_{22}\left(\frac{dt}{dx}\right)^2\right]\frac{dt}{dx} - a_{12}\left(\frac{dt}{dx}\right)^3 dv_t = 0 \tag{23}$$

In difference form the above equation can be written as

$$\Delta u_x\left[1 - a_{22}\left(\frac{dt}{dx}\right)^2\right] + a_{12}\,\Delta v_x\left(\frac{dt}{dx}\right)^2$$

$$- \Delta u_t\left[1 - a_{22}\left(\frac{dt}{dx}\right)^2\right]\frac{dt}{dx} - a_{12}\,\Delta t_t\left(\frac{dt}{dx}\right)^3 = 0 \tag{24}$$

Thus the values of u_x, v_x, u_t and v_t at any unknown point L can be determined by knowing their values at points P, Q, R and S lying on the four characteristics passing through L and then solving 4 simultaneous equations obtained from Eq. 24. Figure 1 shows the characteristic directions.

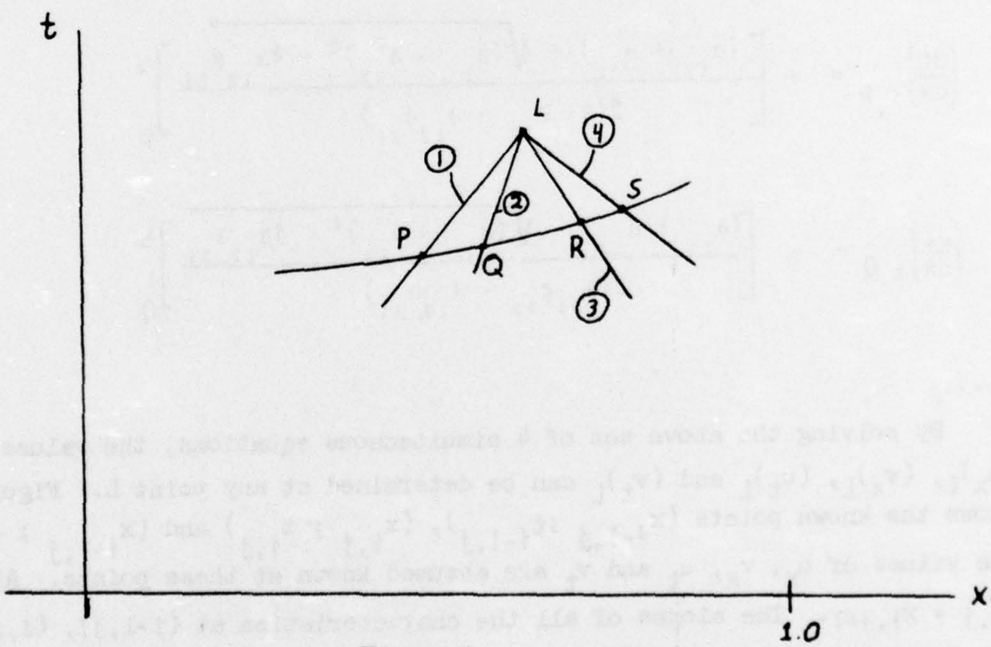


Figure 1

Although the characteristics are curved due to the non-linearity of Eq. 2, it will be assumed that LP, LQ, LR and LS are straight lines; i.e.,

$$\left[(u_x)_L - (u_x)_P\right]\left[1 - (a_{22})_P\left(\frac{dt}{dx}\right)_{1,P}\right] + \left[(v_x)_L - (v_x)_P\right](a_{12})_P\left(\frac{dt}{dx}\right)^2_{1,P}$$

$$- \left[(u_t)_L - (u_t)_P\right]\left[1 - (a_{22})_P\left(\frac{dt}{dx}\right)^2_{1,P}\right]\left(\frac{dt}{dx}\right)_{1,P}$$

$$- (a_{12})_P\left(\frac{dt}{dx}\right)^3_{1,P}\left[(v_t)_L - (v_t)_P\right] = 0 \tag{25}$$

$$\left[(u_x)_L - (u_x)_Q\right]\left[1 - (a_{22})_Q\left(\frac{dt}{dx}\right)_{2,Q}\right] + \left[(v_x)_L - (v_x)_Q\right](a_{12})_Q\left(\frac{dt}{dx}\right)^2_{2,Q}$$

$$- \left[(u_t)_L - (u_t)_Q\right]\left[1 - (a_{22})_Q\left(\frac{dt}{dx}\right)^2_{2,Q}\right]\left(\frac{dt}{dx}\right)_{2,Q}$$

$$- (a_{12})_Q\left(\frac{dt}{dx}\right)^3_{2,Q}\left[(v_t)_L - (v_t)_Q\right] = 0 \tag{26}$$

....

where

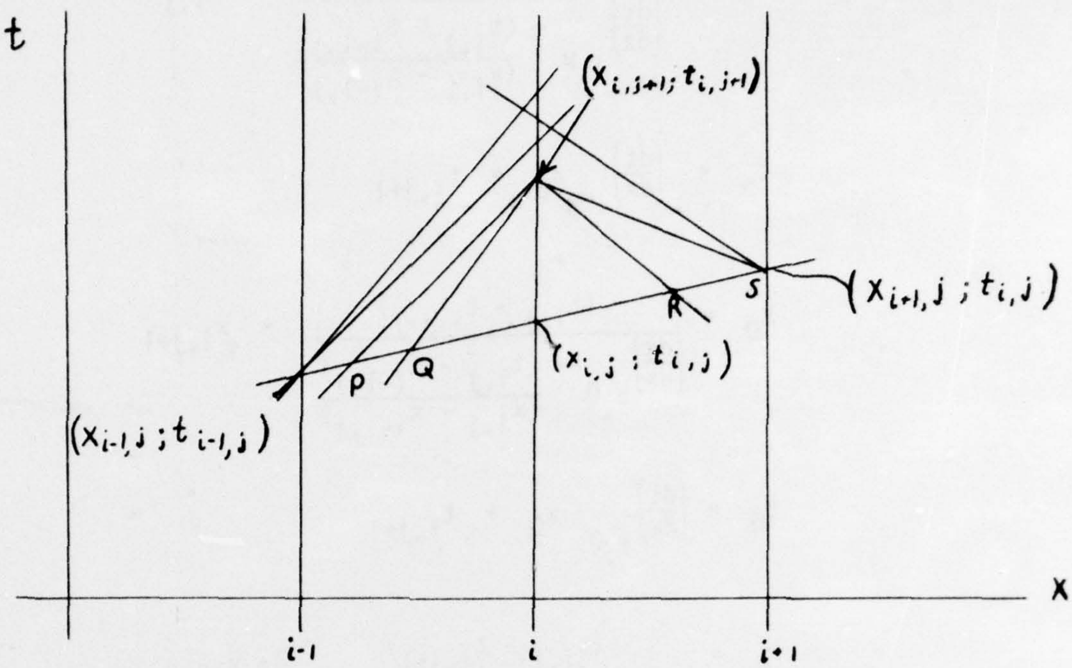
$$\left(\frac{dt}{dx}\right)_{1,P} = + \left[\frac{(a_{11} + a_{22}) + \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}}{2(a_{11}a_{22} - a_{12}a_{21})}\right]^{\frac{1}{2}}_P \tag{27}$$

$$\left(\frac{dt}{dx}\right)_{2,Q} = + \left[\frac{(a_{11} + a_{22}) - \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}}{2(a_{11}a_{22} - a_{12}a_{21})}\right]^{\frac{1}{2}}_Q \tag{28}$$

.....

By solving the above set of 4 simultaneous equations, the values of $(u_x)_L$, $(v_x)_L$, $(u_t)_L$ and $(v_t)_L$ can be determined at any point L. Figure 2 shows the known points $(x_{i-1,j}\ ; t_{i-1,j})$, $(x_{i,j}\ ; t_{i,j})$ and $(x_{i+1,j}\ ; t_{i+1,j})$. The values of u_x, v_x, u_t and v_t are assumed known at these points. Also $x_{i,j} = x_{i,j+1}$. The slopes of all the characteristics at (i-1,j), (i,j) and (i+1,j) are known.

Figure 2



Four different values of $t_{i,j+1}$ are determined by knowing the slope of the characteristic at $(i-1,j)$ and $(i+1,j)$. Hence

$$\left[t_{i,j+1}\right]_1 = \left[x_{i,j+1} - x_{i-1,j}\right]\left(\frac{dt}{dx}\right)_{1(i-1,j)} + t_{i-1,j} \tag{29}$$

$$\left[t_{i,j+1}\right]_2 = \left[x_{i,j+1} - x_{i-1,j}\right]\left(\frac{dt}{dx}\right)_{2(i-1,j)} + t_{i-1,j} \tag{30}$$

$$\left[t_{i,j+1}\right]_3 = \left[x_{i,j+1} - x_{i+1,j}\right]\left(\frac{dt}{dx}\right)_{3(i+1,j)} + t_{i+1,j} \tag{31}$$

and

$$\left[t_{i,j+1}\right]_4 = \left[x_{i,j+1} - x_{i+1,j}\right]\left(\frac{dt}{dx}\right)_{4(i+1,j)} + t_{i+1,j} \tag{32}$$

The smallest value of the above $\left|t_{i,j+1}\right|$'s is taken in order to remain inside the domain of dependence. Once the coordinates of point L $(x_{i,j+1}; t_{i,j+1})$ are known the coordinates of P, Q, R, and S can be computed as follows:

$$x_p = \frac{(t_{i,j} - t_{i,j+1})}{\left(\frac{dt}{dx}\right)_{1,p} - \frac{(t_{i,j} - t_{i-1,j})}{(x_{i,j} - x_{i-1,j})}} + x_{i,j} \tag{33}$$

$$t_p = \left(\frac{dt}{dx}\right)_{1,p} x_p + t_{i,j+1} \tag{34}$$

$$x_Q = \frac{(t_{i,j} - t_{i,j+1})}{\left(\frac{dt}{dx}\right)_{2,Q} - \frac{(t_{i,j} - t_{i-1,j})}{(x_{i,j} - x_{i-1,j})}} + x_{i,j+1} \tag{35}$$

$$t_Q = \left(\frac{dt}{dx}\right)_{2,Q} x_Q + t_{i,j+1} \tag{36}$$

$$x_R = \frac{(t_{i,j} - t_{i,j+1})}{\left(\frac{dt}{dx}\right)_{3,R} - \frac{(t_{i,j} - t_{i+1,j})}{(x_{i,j} - x_{i+1,j})}} + x_{i,j+1} \tag{37}$$

$$t_R = \left(\frac{dt}{dx}\right)_{3,R} x_R + t_{i,j+1} \tag{38}$$

$$x_s = x_{i+1,j} \tag{39}$$

and $$t_s = t_{i+1,j} \tag{40}$$

It will now be assumed that

$$\left(\frac{dt}{dx}\right)_{1,p} \simeq \left(\frac{dt}{dx}\right)_{1,(i-1,j)} \tag{41}$$

$$\left(\frac{dt}{dx}\right)_{2,Q} \simeq \left(\frac{dt}{dx}\right)_{2,(i-1,j)} \tag{42}$$

$$\left(\frac{dt}{dx}\right)_{3,R} \simeq \left(\frac{dt}{dx}\right)_{3,(i+1,j)} \tag{43}$$

and

$$\left(\frac{dt}{dx}\right)_{4,S} \simeq \left(\frac{dt}{dx}\right)_{4,(i+1,j)} \tag{44}$$

The values of the coefficients a_{11}, a_{12}, a_{21} and a_{22} and the values of u_x; v_x, u_t and v_t can be computed at the points P,Q,R and S by linear interpolation. Hence

$$(a_{pq})_P = \frac{\left[(a_{pq})_{i-1,j} - (a_{pq})_{i,j}\right](x_P - x_{i,j})}{(x_{i-1,j} - x_{i,j})} + (a_{pq})_{i,j} \tag{45}$$

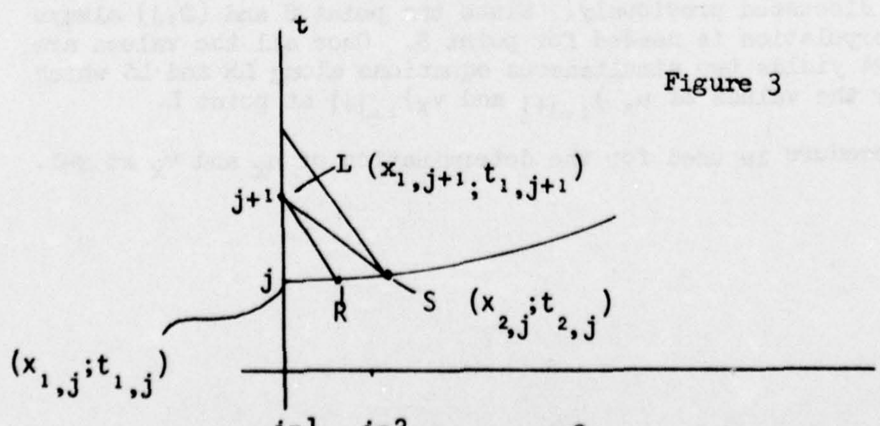
.....

Once the values of the coefficients a_{ij} and the dependent variables u_x, v_x, u_t and v_t are known at P, Q, R and S, then the simultaneous equations obtained from Eq. 24 can be solved and thus yield the values of $u_x)_L$, $v_x)_L$ $u_t)_L$ and $v_t)_L$ at the point L. Once these values are known at each nodal point for a new time step j+1, movement to the next step is determined by the marching forward procedure.

The values of the dependent variables can be computed for any nodal point other than the end points. The above method must be slightly modified in order to compute the dependent variables at the end points.

At both the ends (x=0 and x=1), u_t and v_t are given for all values of time and the only unknowns are u_x and v_x at these end points. In order to determine the values of u_x and v_x for the end points at the new time step (j+1) use is made of only two characteristics. For a determination of the values of u_x and v_x at x=0 (i=1) at point L, use is made of the characteristics of family 3 and 4 as shown in Fig. 3.

Figure 3



Then by knowing the slopes of the characteristics at (2,j), two different values of $t_{1,j+1}$ are determined. Hence

$$\left|t_{1,j+1}\right|_1 = \left|x_{1,j+1} - x_{2,j}\right|\left(\frac{dt}{dx}\right)_{3(2,j)} + t_{2,j} \tag{46}$$

$$\left|t_{1,j+1}\right|_2 = \left|x_{1,j+1} - x_{2,j}\right|\left(\frac{dt}{dx}\right)_{4(2,j)} + t_{2,j} \tag{47}$$

In order to remain inside the domain of dependence the smaller of the above two values is taken as the value of $t_{1,j+1}$ for any further computation. Hence

$$x_R = \frac{(t_{1,j} - t_{1,j+1})}{\left(\frac{dt}{dx}\right)_{3,R} - \frac{(t_{1,j} - t_{2,j})}{(x_{1,j} - x_{2,j})}} + x_{1,j+1} \tag{48}$$

$$t_R = \left(\frac{dt}{dx}\right)_{3,R} x_R + t_{1,j+1} \tag{49}$$

$$x_S = x_{2,j} \tag{50}$$

and

$$t_S = t_{2,j} \tag{51}$$

It is assumed that

$$\left.\frac{dt}{dx}\right)_{3,R} \simeq \left(\frac{dt}{dx}\right)_{3(2,j)} \tag{52}$$

Now the values of the coefficients a_{11}, a_{12}, a_{21} and a_{22} and the values of u_x, v_x, u_t and v_t can be computed at the point R by the methed of linear interpolation as discussed previously. Since the point S and (2,j) always coincide, no interpolation is needed for point S. Once all the values are known at R, Eq. 24 yields two simultaneous equations along LR and LS which can be solved for the values of $\left.u_x\right)_{1,j+1}$ and $\left.v_x\right)_{1,j+1}$ at point L.

A similar procedure is used for the determination of u_x and v_x at x=1.

FINITE DIFFERENCE METHOD

Equations 11, 12, 13 and 14 can be written in finite difference form for any grid point (i,j) as follows:

$$(a_{11})_{i,j}\left(\frac{\partial u_x}{\partial x}\right)_{i,j} + (a_{12})_{i,j}\left(\frac{\partial v_x}{\partial x}\right)_{i,j} - \left(\frac{\partial u_t}{\partial t}\right)_{i,j} = 0 \qquad (53)$$

$$(a_{21})_{i,j}\left(\frac{\partial u_x}{\partial x}\right)_{i,j} + (a_{22})_{i,j}\left(\frac{\partial v_x}{\partial x}\right)_{i,j} - \left(\frac{\partial v_t}{\partial t}\right)_{i,j} = 0 \qquad (54)$$

$$\left(\frac{\partial u_x}{\partial t}\right)_{i,j+1} - \left(\frac{\partial u_t}{\partial x}\right)_{i,j+1} = 0 \qquad (55)$$

$$\left(\frac{\partial v_x}{\partial t}\right)_{i,j+1} - \left(\frac{\partial v_t}{\partial x}\right)_{i,j+1} = 0$$

where i= 1,2,3,...n,n+1 (Fig.4).

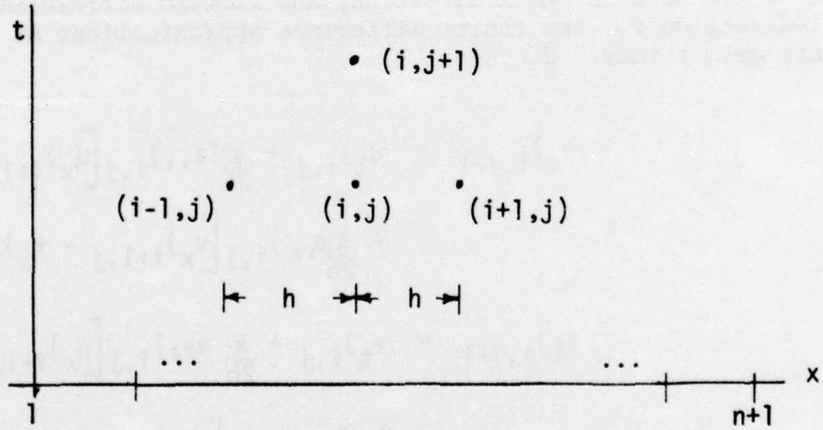


Figure 4

The initial conditions are

$$u_x)_{i,1} = f_1(x) \qquad v_x)_{i,1} = f_2(x)$$

$$u_t)_{i,1} = 0 \qquad v_t)_{i,1} = 0$$

and the boundary conditions are

$$u_t)_{1,j} = \phi_1(t) \qquad u_t)_{n+1,j} = \phi_2(t)$$

$$v_t)_{1,j} = \Psi_1(t) \qquad v_t)_{n+1,j} = \Psi_2(t)$$

Since the x axis is divided into n parts, the distance between two consecutive grid points is

$$x_i - x_{i-1} = h = \frac{1}{n}$$

In order to remain inside the domain of dependence

$$k/n \ngtr \left.\frac{dt}{dx}\right)_{i,j} \quad \text{or} \quad k \ngtr h \left[\frac{(a_{11} + a_{22}) - \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}}{2(a_{11}a_{22} - a_{12}a_{21})}\right]^{\frac{1}{2}}$$

This is the Von Newman stability criteria for finite difference approximations of hyperbolic equations. This must be checked at each grid point, before carring out any further computations. Central difference approximations are used in the x direction, and forward differences are used in the t-direction for the finite difference approximations at all intermediate grid points. Hence

$$(u_t)_{i,j+1} = u_t)_{i,j} + \frac{k}{2h}(a_{11})_{i,j}\left[u_x)_{i+1,j} - u_x)_{i-1,j}\right] + \frac{k}{2h}a_{12})_{i,j}\left[v_x)_{i+1,j} - v_x)_{i-1,j}\right]$$

$$v_t)_{i,j+1} = v_t)_{i,j} + \frac{k}{2h}\, a_{21})_{i,j}\left[u_x)_{i+1,j} - u_x)_{i-1,j}\right] + \frac{k}{2h}a_{22})_{i,j}\left[v_x)_{i+1,j} - v_x)_{i-1,j}\right]$$

$$u_x)_{i,j+1} = u_x)_{i,j} + \frac{k}{2h}\left[u_t)_{i+1,j+1} - u_t)_{i-1,j+1}\right]$$

$$v_x)_{i,j+1} = v_x)_{i,j} + \frac{k}{2h}\left[v_t)_{i+1,j+1} - v_t)_{i-1,j+1}\right]$$

Once the values of u_t, v_t, u_x and v_x have been computed at all the intermediate points by the above formulas, the values of u_x, v_x can be computed at the two end grid points (i=1, and i=n+1) by forward and backward-differences respectively as

(a) at (i=1)

$$u_x)_{1,j+1} = u_x)_{1,j} + \frac{k}{2h}\Big[-3u_t)_{1,j+1} \quad 4u_t)_{2,j+1} - u_t)_{3,j+1}\Big]$$

$$v_x)_{1,j+1} = v_x)_{1,j} + \frac{k}{2h}\Big[-3v_t)_{1,j+1} + 4v_t)_{2,j+1} - v_t)_{3,j+1}\Big]$$

and

(b) at (i=n+1)

$$u_x)_{n+1,j+1} = u_x)_{n+1,j} + \frac{k}{2h}\Big[u_t)_{n-1,j+1} - 4u_t)_{n,j+1} + 3u_t)_{n+1,j+1}\Big]$$

$$v_x)_{n+1,j+1} = v_x)_{n+1,j} + \frac{k}{2h}\Big[v_t)_{n-1,j+1} - 4v_t)_{n,j+1} + 3v_t)_{n+1,j+1}\Big]$$

Thus the values of u_x, v_x, u_t, and v_t can be computed at all the nodal points and by the similar Marching-Forward procedure can be computed for any time step.

NUMERICAL CALCULATION

Phillips and Costello [16] have derived the governing equations for the large deflections of helical springs in which

$$a_{11} = (v_x \sin\alpha + \cos\alpha)\sin\alpha \left\{-\frac{\nu}{1+\nu}(v_x \sin\alpha + \cos\alpha) + \frac{\cos^2\alpha}{\left[1-(1+u_x)^2\sin^2\alpha\right]^{3/2}}\right\}$$

$$a_{12} = a_{21} = \sin^2\alpha \left\{\frac{(1+u_x)\cos^2\alpha}{\left[1-(1+u_x)^2\sin^2\alpha\right]^{1/2}} - \frac{\cos\alpha}{(1+\nu)} - \frac{2\nu}{(1+\nu)}(1+u_x)(v_x\sin\alpha + \cos\alpha)\right\}$$

$$a_{22} = \sin\alpha\left\{1 - \frac{\nu}{1+\nu}(1+u_x)^2\sin^2\alpha\right\}$$

The spring for which the numerical values have been computed has the following specifications

Number of coils = 3

helix angle α = 0.141815 radians

original length of spring L = 19.0 in

original mean coil radius R = 7.06 in

wire radius r = 0.594 in

Poisson's ratio ν = 0.29

mass spring = 0.1093514367

modulus of elasticity E = 30×10^6 psi

initial compression Δ = 6.5 in

axial strain $\epsilon = u_x$

rotational strain $\beta = v_x$

initial conditions are $u_x(x,0) = -\frac{\Delta}{L}$, $v_x(x,0) = 0$

$u_t(x,0) = 0 \quad v_t(x,0) = 0$

the boundary conditions are

$u_t(0,t) = \phi_1(t)$ given (velocity of the impacted end x = 0)

$u_t(o,t) = 0 \qquad u_t(1,t) = 0 \qquad v_t(1,t) = 0$

The velocity of the impacted end $u_t(0,t) = \phi_1(t)$ is shown in Fig. 5.

The results of the method of characteristics are shown in Fig. 6, 7, 8, 9, 10. The axial strain, rotational strain, axial force and axial moment are shown at the impacted end x = 0.

The results of the method of finite differences are shown in Fig. 11, 12, 13, 14. The axial strain, rotational strain, axial force and axial moment are shown at the impacted end x = 0.

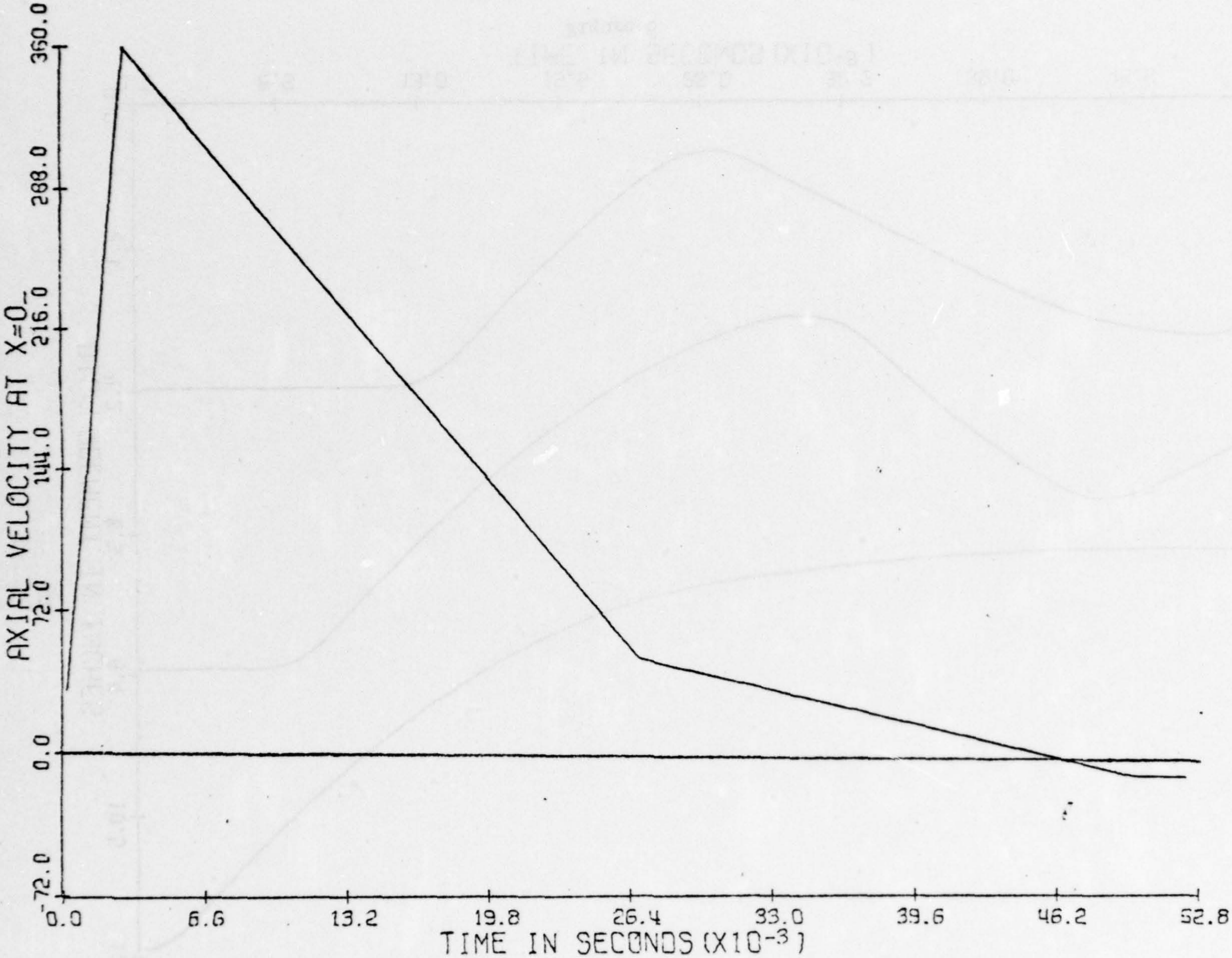
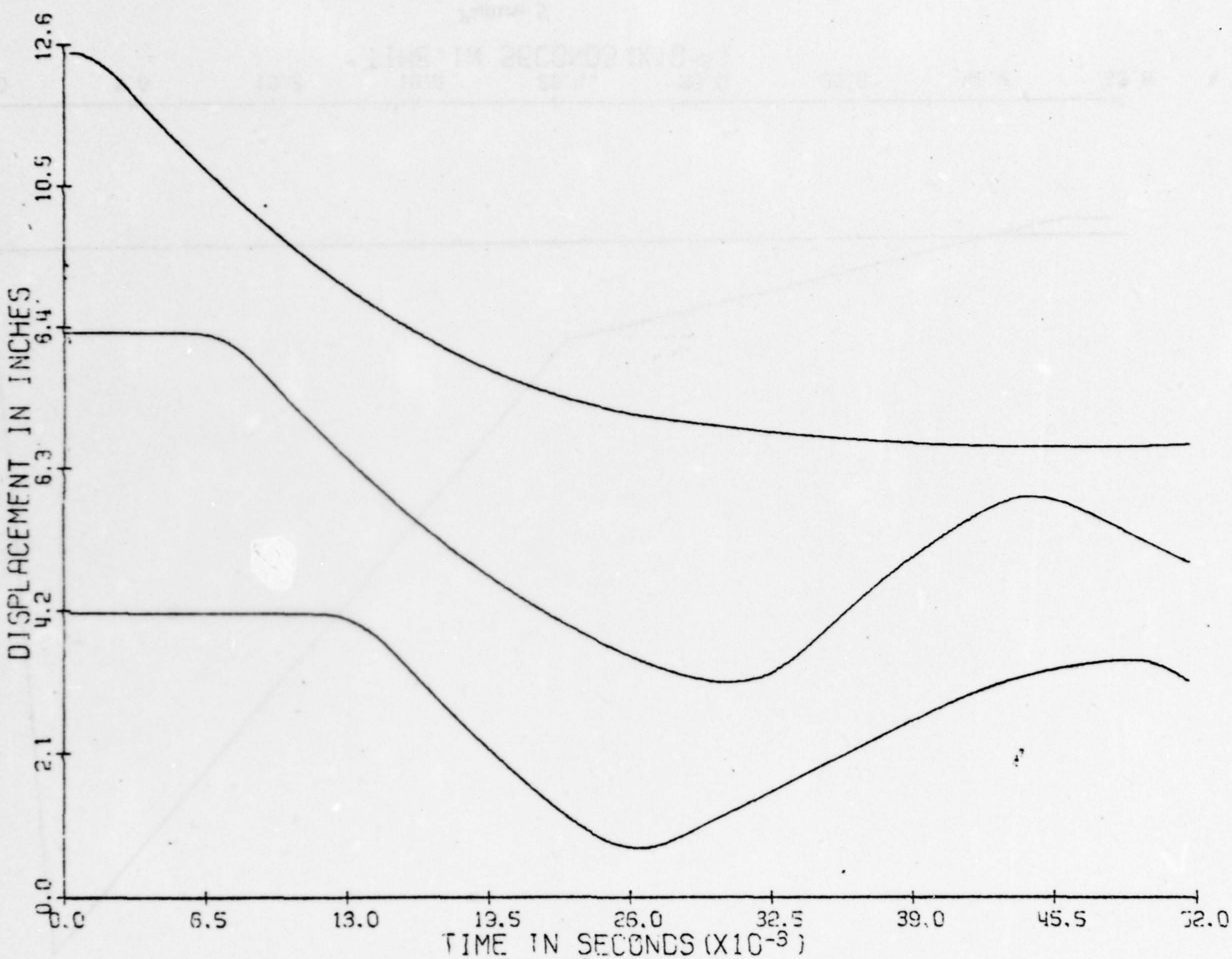
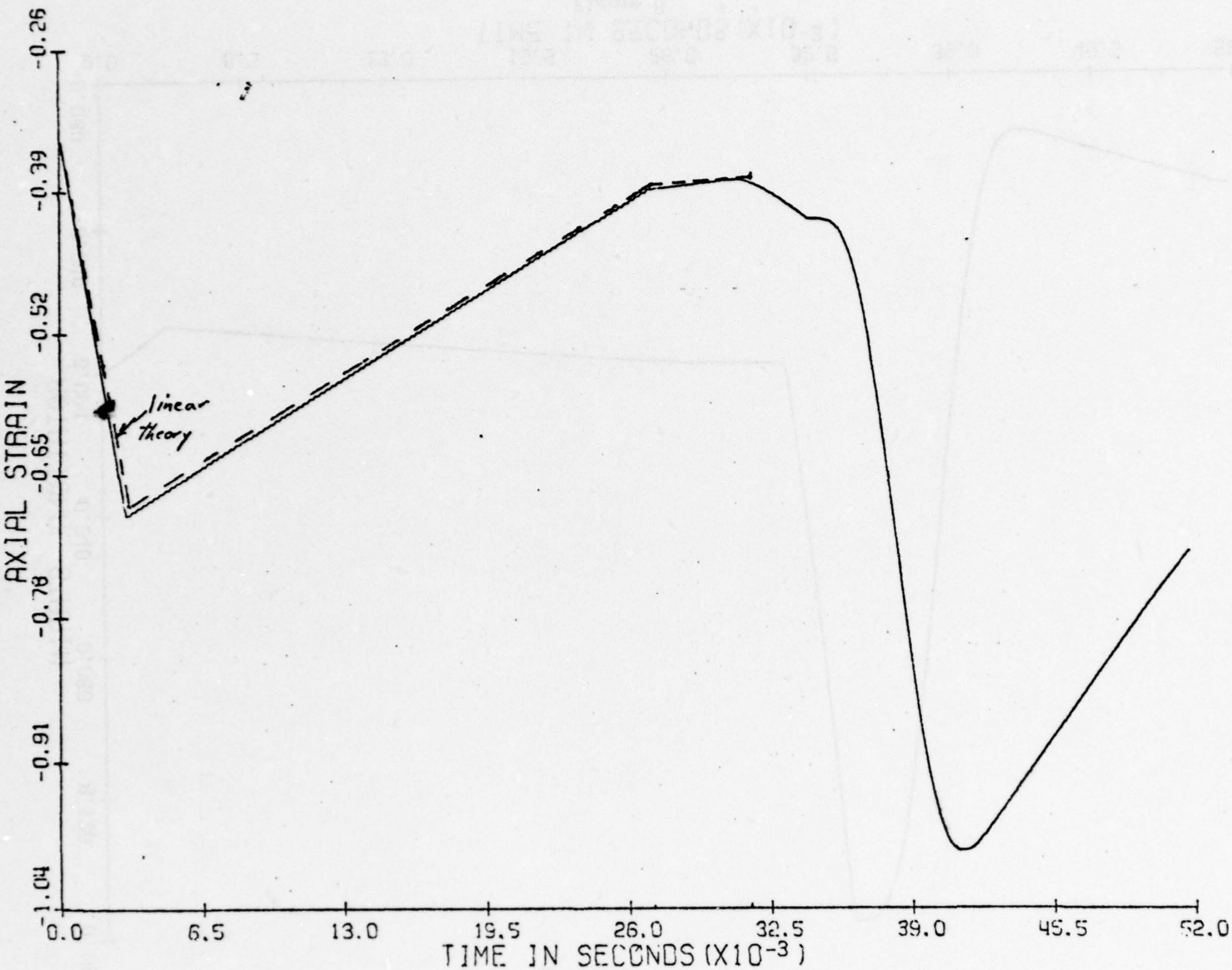

AXIAL VELOCITY AT X=0_
360.0
288.0
216.0
144.0
72.0
0.0
-72.0
0.0
6.6
13.2
19.8
26.4
33.0
39.6
46.2
52.8
TIME IN SECONDS (X10^-3)

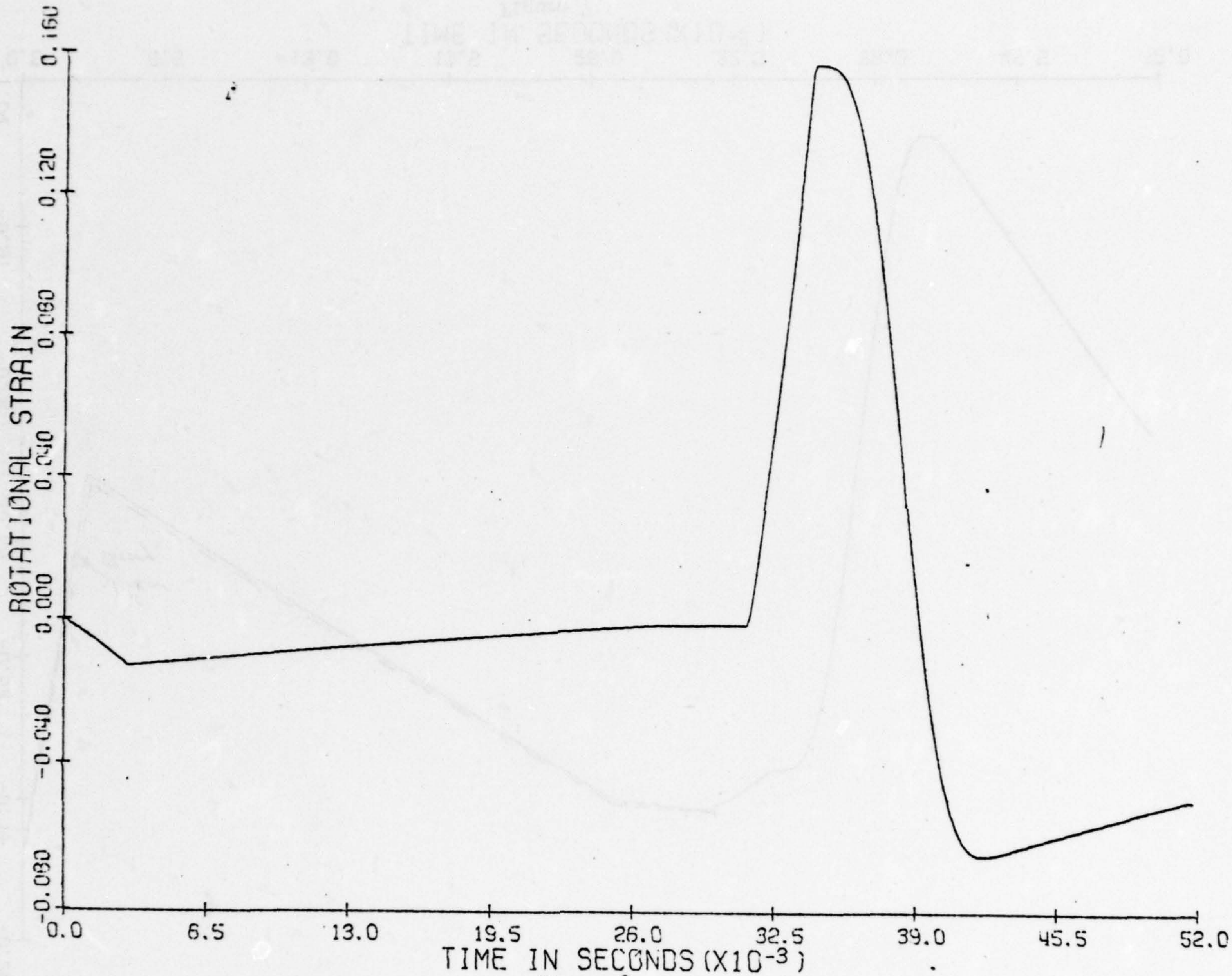
Figure 5



12.6
10.5
8.4
6.3
4.2
2.1
0.0
DISPLACEMENT IN INCHES
0.0
6.5
13.0
19.5
26.0
32.5
39.0
45.5
52.0
TIME IN SECONDS (X10^{-3})

Figure 6


AXIAL STRAIN
-0.26
-0.39
-0.52
-0.65
-0.78
-0.91
-1.04
0.0
6.5
13.0
19.5
26.0
32.5
39.0
45.5
52.0
TIME IN SECONDS (X10^-3)
linear theory
Figure 7


0.160
0.120
0.080
0.040
0.000
-0.040
-0.080
ROTATIONAL STRAIN
0.0
6.5
13.0
19.5
26.0
32.5
39.0
45.5
52.0
TIME IN SECONDS (X10^-3)
Figure 8

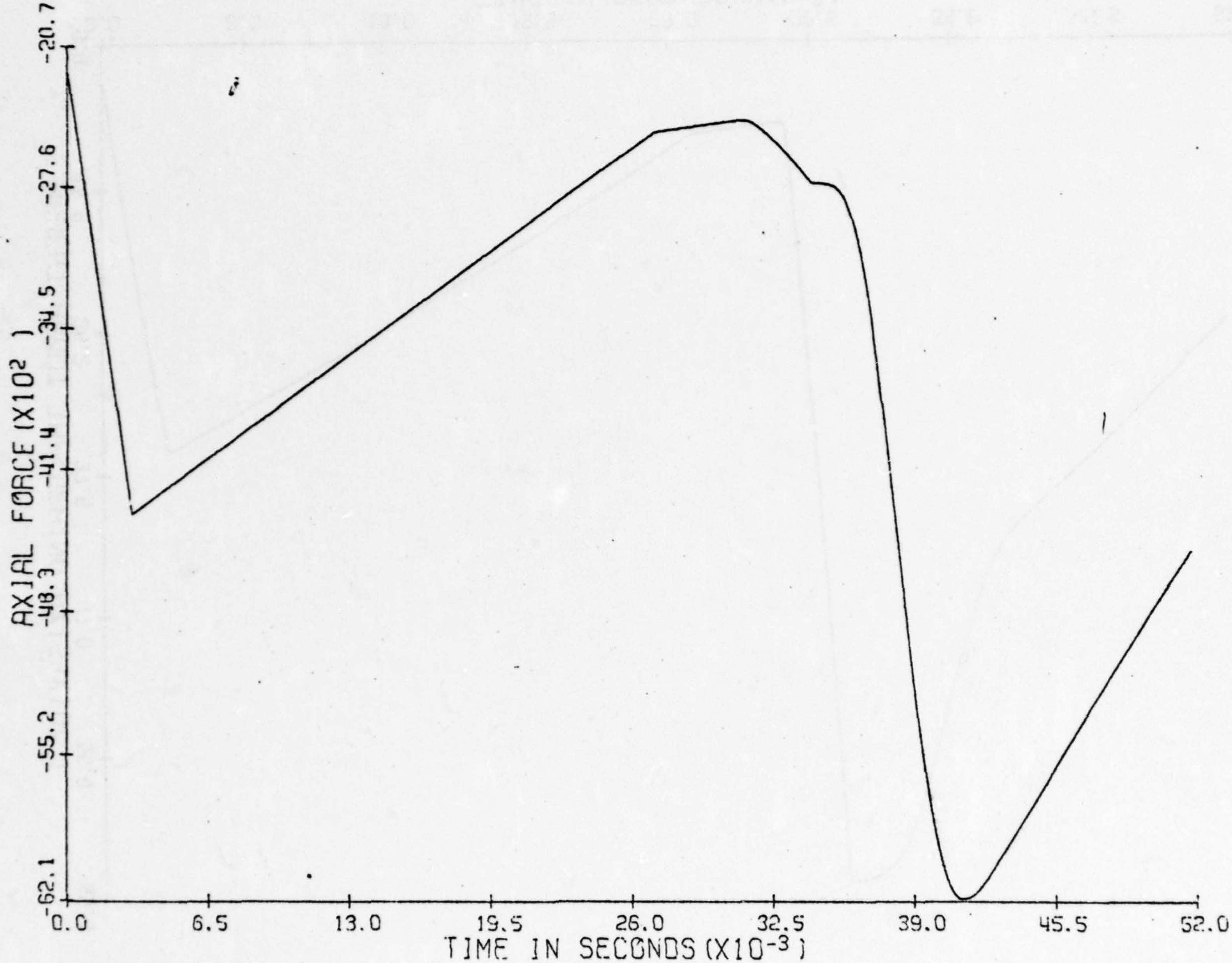
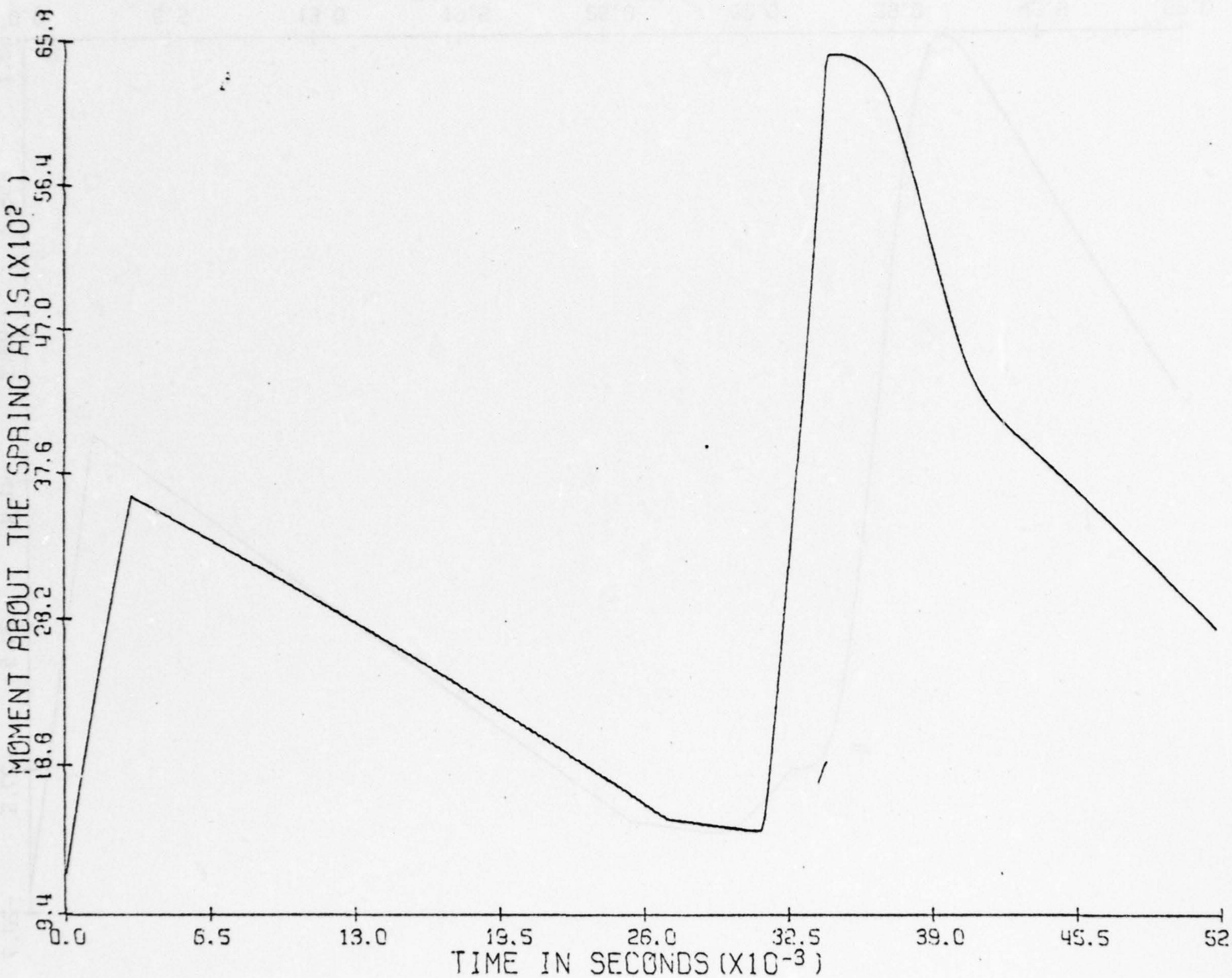

AXIAL FORCE (X10^2)
-20.7
-27.6
-34.5
-41.4
-48.3
-55.2
-62.1
0.0
6.5
13.0
19.5
26.0
32.5
39.0
45.5
52.0
TIME IN SECONDS (X10^{-3})

Figure 9



MOMENT ABOUT THE SPRING AXIS (X10²)
65.8
56.4
47.0
37.6
28.2
18.8
9.4
0.0
6.5
13.0
19.5
26.0
32.5
39.0
45.5
52.0
TIME IN SECONDS (X10⁻³)

Figure 10

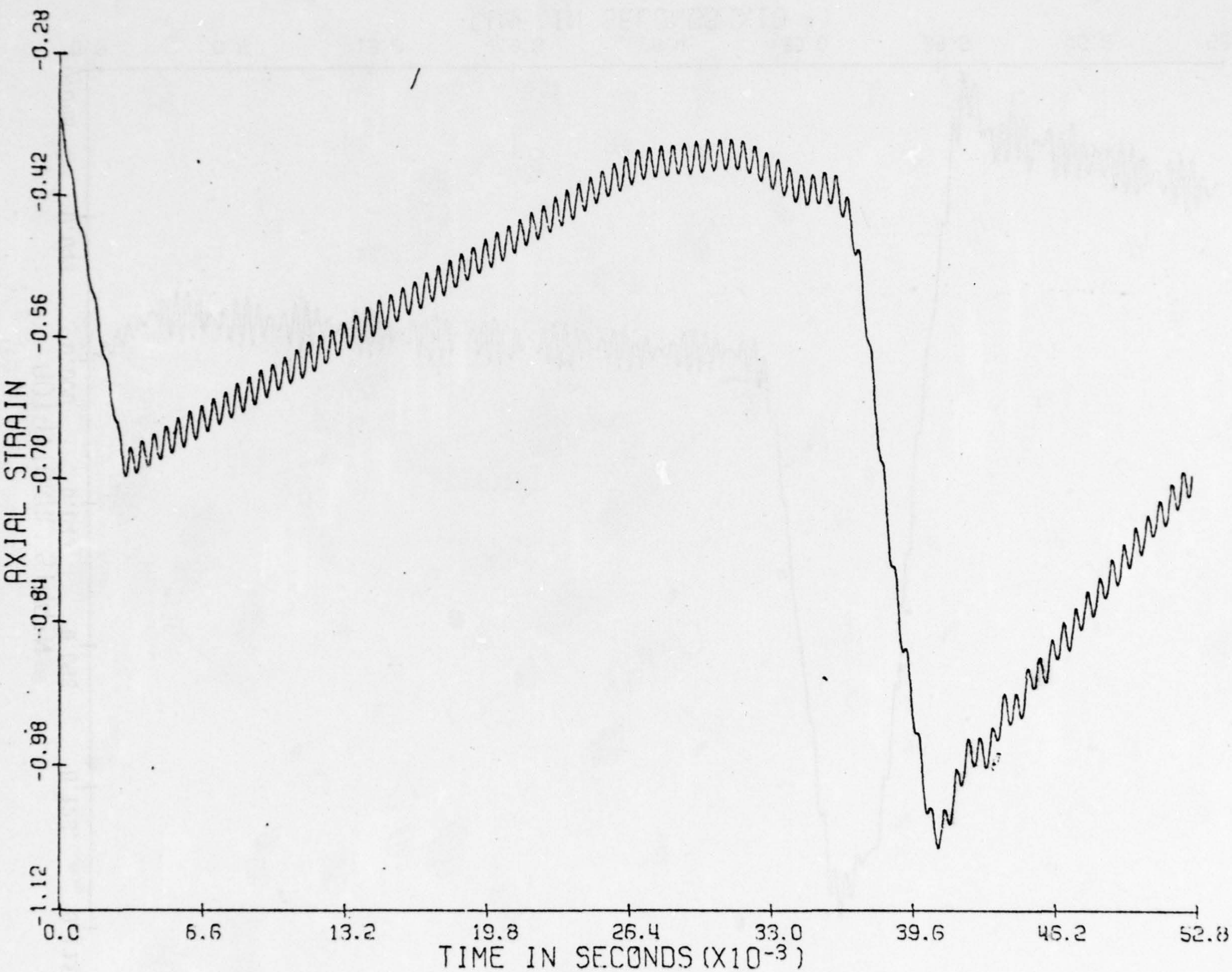

AXIAL STRAIN
-0.28
-0.42
-0.56
-0.70
-0.84
-0.98
-1.12
0.0
6.6
13.2
19.8
26.4
33.0
39.6
46.2
52.8
TIME IN SECONDS (X10^{-3})

Figure 11

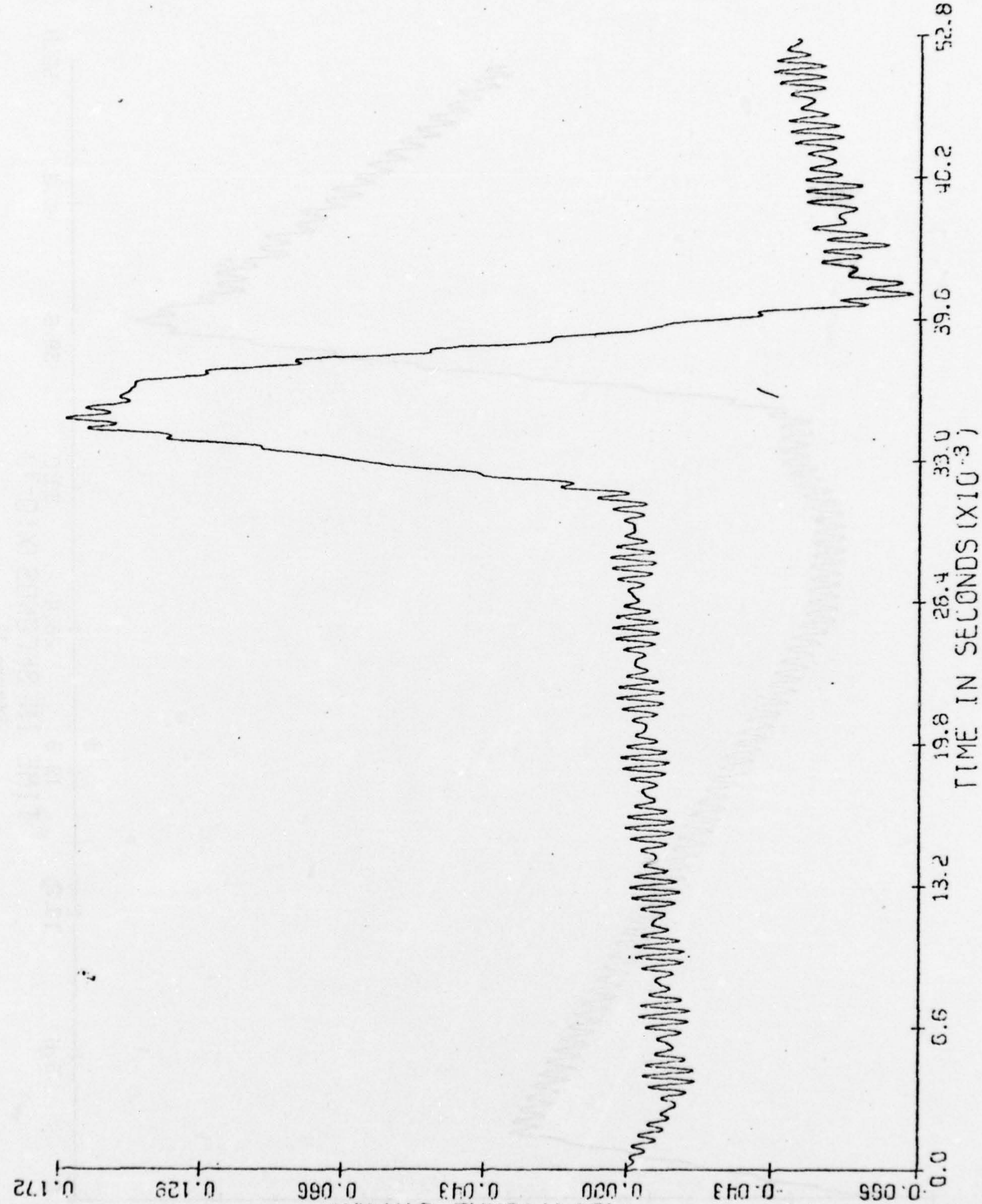
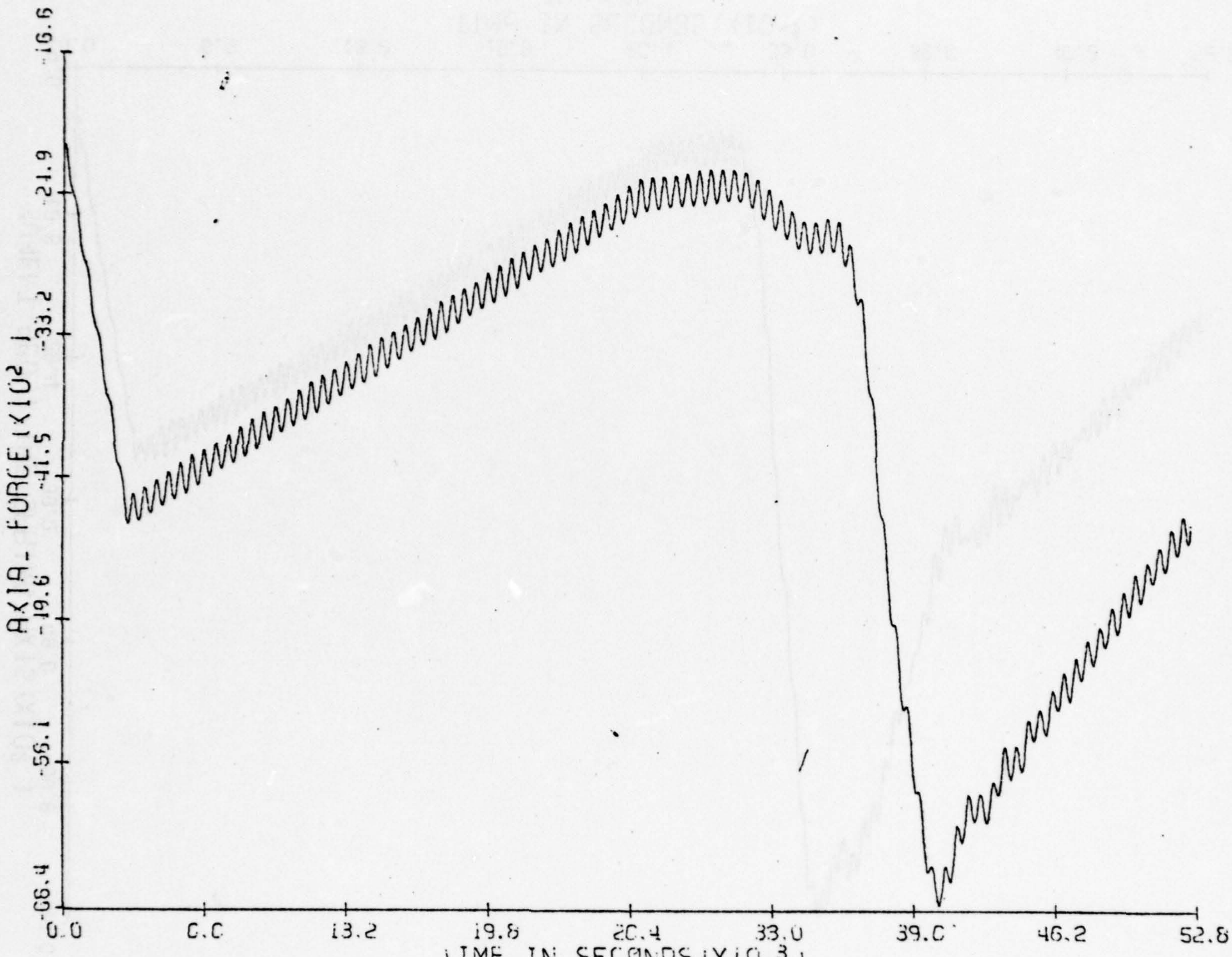
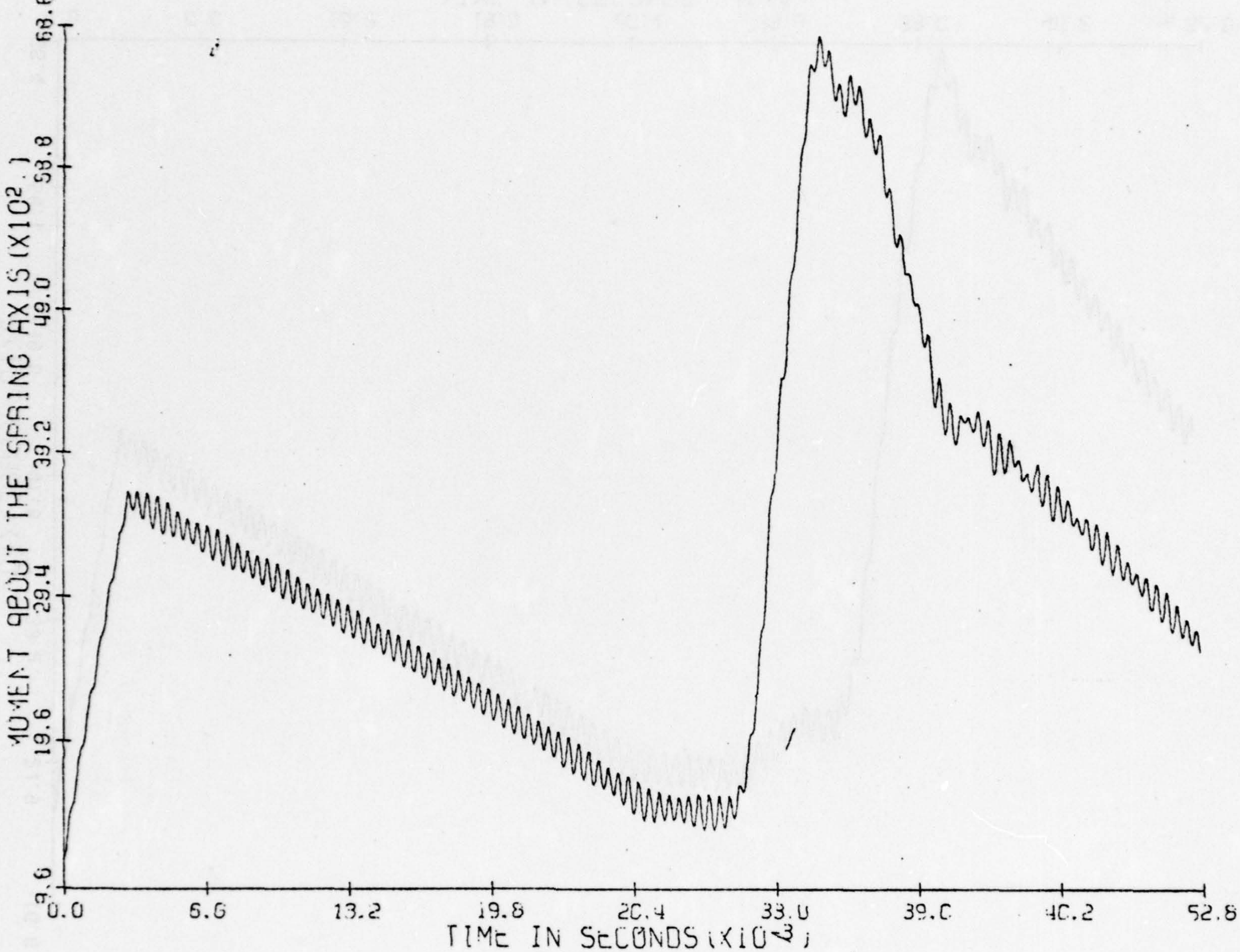

ROTATIONAL STRAIN
0.172
0.129
0.086
0.043
0.000
-0.043
-0.086
TIME IN SECONDS (X10^-3)
0.0
6.6
13.2
19.8
26.4
33.0
39.6
46.2
52.8

Figure 12



-16.6
-21.9
-33.2
-41.5
-49.6
-56.1
-68.4
AXIA- FORCE (X10²)
0.0
6.6
13.2
19.8
26.4
33.0
39.6
46.2
52.8
TIME IN SECONDS (X10 ³)

Figure 13



MOMENT ABOUT THE SPRING AXIS (X10².)
68.6
58.8
49.0
39.2
29.4
19.6
9.6
0.0
6.6
13.2
19.8
20.4
33.0
39.0
40.2
52.8
TIME IN SECONDS (X10⁻³)

Figure 14

SUMMARY AND CONCLUSIONS

In this report the dynamic non-linear equations of motion for an impacted helical spring are solved numerically using the method of characteristics and the method of finite differences. Both results compare favorably. The non-linear equations of motion for the spring are presented in the attached paper[16]. Also attached to this report is a paper [17] which presents a method for determining the radial expansion of the spring.

The results in this report and the attached paper [17] indicate that when a spring is subjected to an axial impact with an end velocity diagram similar to that shown in Fig. 5, significant torsional oscillations are set up in the spring. The results also show that at the impacted end, for a given time, the normal axial force can be relatively small while the axial twisting moment can be rather large. Under such conditions it is possible for one end of the spring to slip and rotate relative to the other end. This rotation of one end relative to the other end may cause the spring to wind up on the inside cylinder surrounding the spring. If this occurs as the spring is in a period of large compression, it is possible for the spring to tighten up on the inside cylinder as the spring tends to expand. This tightening up of the spring would introduce rather large frictional forces on the inside of the spring and hence would reduce the springs capacity to expand. This decrease could result in a possible hang-out-of-battery. It is also possible for the spring to slip at the ends so that the spring contacts the outside cylinder. The frictional forces on the outside of the spring may also be sufficient to cause hang-out-of-battery.

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Reprinted from the Journal of Applied Mechanics,
Volume 42, Number 4, Pages 789-792, December 1975

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Radial Expansion of Impacted Helical Springs

A theory is presented which will predict the radial expansion of impacted helical springs. This becomes important in cases where the spring is surrounded by a cylindrical constraint because of the possibility of excessive wear. The results show that the dynamic radial expansion can be much larger than that predicted by statical methods.

Introduction

Helical springs are used in many cases to resist impact loads and, in certain instances, the springs resisting such loads are surrounded by cylindrical constraints. Since the velocity of the impacted end may be rather large, the possibility of excessive wear exists if the spring comes in contact with the cylindrical constraints. The dynamic radial expansion therefore becomes an important factor in the design of such a spring.

Love [1],[1] presents expressions for the static response of helical springs subjected to large deflections. The dynamic response of springs is treated in articles by Johnson [2], Dick [3], Krebs and Weidlich [4], Geballe [5], Kagawa [6], Britton and Langley [7], Johnson and Stewart [8], Wittrick [9] and Durant [10]. The foregoing authors, with the exception of Love, have restricted their analysis to small displacements about an equilibrium position.

In an article by Phillips and Costello [11], a theoretical and experimental investigation is made of the large deflections of impacted springs. Stokes [12], in a recent paper, conducted an analytical and experimental program to investigate the radial expansion of helical springs due to longitudinal impact. Of the 10 tests conducted only one set of photographic results was sharp enough to obtain meaningful data on expansion. Also, Stokes indicates that his minimum complexity model, "does not take end effects into account and therefore, strictly speaking, applies only to an infinite spring." It is felt by this author, after observing a film of the dynamic impact of a helical spring, that the end effects are very important. Significant torsional oscillations can occur in the spring if one end of the spring is fixed: a result of the reflections from the fixed end. It is the purpose of this paper to present an expression for determining the radial expansion of impacted springs.

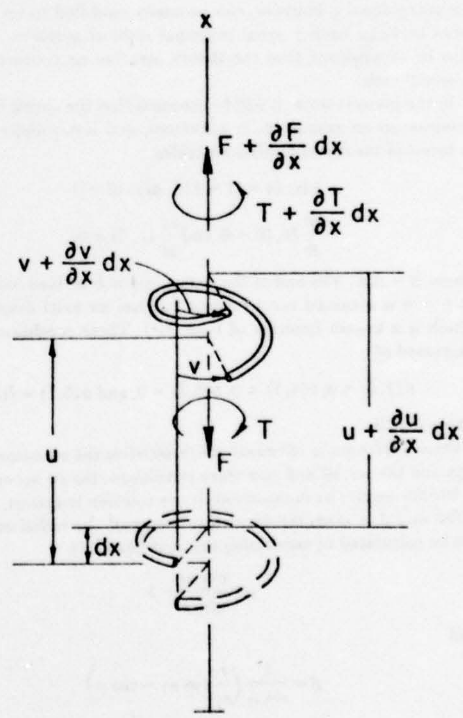

Fig. 1 Typical spring element

Theory

A consideration of the variations of the axial force F and the axial torque T leads to the equations of motion for a typical spring element, Fig. 1. The resulting equations are derived in [11] and are presented here for further discussion. A summation of force in the x-direction yields

$$a \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + b \frac{\partial^2 \bar{\theta}}{\partial \bar{x}^2} = \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} \tag{1}$$

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[1] Numbers in brackets designate References at end of paper.

Contributed by the Applied Mechanics Division and presented at the Winter Annual Meeting, Houston, Texas, November 30–December 4, 1975, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion on this paper should be addressed to the Editorial Department, ASME, United Engineering Center, 345 East 47th Street, New York, N. Y. 10017, and will be accepted until February 20, 1976. Discussion received after this date will be returned. Manuscript received by ASME Applied Mechanics Division, February, 1975; final revision, June, 1975. Paper No. 75-WA/APM-25.

while a summation of moments about the x-axis yields

$$b\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+c\frac{\partial^2\bar{v}}{\partial\bar{x}^2}=\frac{\partial^2\bar{v}}{\partial\bar{t}^2} \tag{2}$$

where

$$\bar{x}=\frac{x}{h},\ \bar{r}=\frac{r}{h},\ \bar{u}=\frac{u}{h},\ \bar{v}=\frac{v}{h}=\bar{r}\phi,\ \bar{t}=t/(Mhr^2/EI)^{1/2}, \tag{3}$$

$$a=\frac{r^2}{EI}\frac{\partial F}{\partial\epsilon}=\left(1-\frac{\nu}{1+\nu}\cos^2\alpha\right)\sin\alpha \tag{4}$$

$$b=\frac{r^2}{EI}\frac{\partial F}{\partial\beta}=\frac{r}{EI}\frac{\partial T}{\partial\epsilon}=-\frac{\nu}{1+\nu}\sin^2\alpha\cos\alpha \tag{5}$$

$$c=\frac{r}{EI}\frac{\partial T}{\partial\beta}=\left(1-\frac{\nu}{1+\nu}\sin^2\alpha\right)\sin\alpha \tag{6}$$

and x is the axial coordinate, u is the axial displacement of the spring element, t is time, M is the total mass of the spring, ϕ is the rotation of the spring element about the x-axis, r is the radius of the spring helix in the unstretched position, h is the length of the spring in the unstretched position, ϵ is the extensional strain $\partial u/\partial x$, β is the rotational strain $r(\partial\phi/\partial x)$, E is Young's modulus of the spring material, ν is Poisson's ratio of the spring material, α is the angle which the tangent to the helix makes with a plane perpendicular to the axis of the helix in the unstretched position and I is the moment of inertia of the circular cross section. It is assumed in the foregoing equations that the wire cross section is circular. The foregoing theory, however, can be easily modified to include wire cross sections having equal principal radii of gyration. It should also be emphasized that the theory assumes no contact between adjacent coils.

In the present work, it will be assumed that the spring is initially compressed an amount Δ, is untwisted, and is motionless. Hence, in terms of the dimensionless variables

$$\bar{u}(\bar{x},0)=(1-\bar{x})\bar{\Delta},\ \bar{v}(\bar{x},0)=0,$$

$$\frac{\partial\bar{u}}{\partial\bar{t}}(\bar{x},0)=0,\text{ and }\frac{\partial\bar{v}}{\partial\bar{t}}(\bar{x},0)=0, \tag{7}$$

where $\bar{\Delta}=\Delta/h$. The end of the spring at $x=h$ is fixed and the end at $x=0$ is assumed not to rotate but has an axial displacement which is a known function of time, $f(t)$. These conditions can be expressed as

$$\bar{u}(1,\bar{t})=0,\ \bar{v}(1,\bar{t})=0,\ \bar{v}(0,\bar{t})=0,\text{ and }\bar{u}(0,\bar{t})=\bar{f}(\bar{t}) \tag{8}$$

where $\bar{f}=f/h$.

Once a solution is obtained which satisfies the equations of motion and the initial and boundary conditions, the radial expansion of the spring can be determined. If the solution is known, then $\epsilon=\partial u/\partial x$ and $\beta=r(\partial\phi/\partial x)$ are determined and the radial expansion can be calculated by combining the equations [11]

$$\epsilon=\frac{\sin\alpha_1}{\sin\alpha}-1 \tag{9}$$

and

$$\beta=\frac{1}{\sin\alpha}\left(\frac{r}{r_1}\cos\alpha_1-\cos\alpha\right) \tag{10}$$

where r_1 is the radius of the spring in the stretched position and α_1 is the angle which the tangent to the helix makes with a plane perpendicular to the axis of the helix in the stretched position.

Solution

Equations (1) and (2) become, after taking the Laplace transform

$$a\frac{\partial^2\tilde{u}}{\partial\bar{x}^2}+b\frac{\partial^2\tilde{v}}{\partial\bar{x}^2}=s^2\tilde{u}-s(1-\bar{x})\bar{\Delta} \tag{11}$$

$$b\frac{\partial^2\tilde{u}}{\partial\bar{x}^2}+c\frac{\partial^2\tilde{v}}{\partial\bar{x}^2}=s^2\tilde{v} \tag{12}$$

where

$$\tilde{u}(\bar{x},s)=\mathcal{L}(\bar{u})=\int_0^\infty\bar{u}(\bar{x},\bar{t})e^{-s\bar{t}}d\bar{t} \tag{13}$$

The homogeneous solution of equations (11) and (12) is obtained by assuming a solution of the form

$$\tilde{u}=c_k e^{r_k\bar{x}}\text{ and }\tilde{v}=d_k e^{r_k\bar{x}} \tag{14}$$

A substitution of equation (14) into equations (11) and (12) (the homogeneous equations) yields

$$r_k^2=\frac{(a+c)s^2\pm\sqrt{(a+c)^2s^4-4(ac-b^2)s^4}}{2(ac-b^2)} \tag{15}$$

and

$$d_k=\frac{(s^2-ar_k^2)}{br_k^2}c_k. \tag{16}$$

The particular solution is

$$\tilde{u}_p=\frac{\bar{\Delta}}{s}(1-\bar{x})\text{ and }\tilde{v}_p=0. \tag{17}$$

Equation (15) yields four roots and hence the solution for the transformed variables can be written as

$$\tilde{u}(\bar{x},s)=c_1(s)e^{e_1s\bar{x}}+c_2(s)e^{-e_1s\bar{x}}+c_3(s)e^{e_2s\bar{x}}$$
$$+c_4(s)e^{-e_2s\bar{x}}+\frac{\bar{\Delta}}{s}(1-\bar{x}) \tag{18}$$

and

$$\tilde{v}(\bar{x},s)=d_1(s)e^{e_1s\bar{x}}+d_2(s)e^{-e_1s\bar{x}}+d_3(s)e^{e_2s\bar{x}}+d_4(s)e^{-e_2s\bar{x}} \tag{19}$$

where

$$e_1=\frac{1}{\sqrt{\sin\alpha}}\text{ and }e_2=\sqrt{(1+\nu)/\sin\alpha} \tag{20}$$

Equation (16) yields

$$d_1=g_1c_1,\ d_2=g_1c_1,\ d_3=g_2c_3,\text{ and }d_4=g_2c_4 \tag{21}$$

where

$$g_1=\frac{(\sin\alpha-a)}{b}\text{ and }g_2=\left(\frac{\sin\alpha}{1+\nu}-a\right)\frac{1}{b}. \tag{22}$$

A satisfaction of the transformed boundary conditions

$$\tilde{v}(0,s)=\tilde{v}(1,s)=\tilde{u}(1,s)=0,\text{ and }\tilde{u}(0,s)=\tilde{f}(s) \tag{23}$$

results in

$$c_1(s)=-\frac{g_2}{g_2-g_1}\left(\tilde{f}(s)-\frac{\bar{\Delta}}{s}\right)\frac{e^{-e_1s}}{e^{e_1s}-e^{-e_1s}}$$

$$c_2(s)=\frac{g_2}{g_2-g_1}\left(\tilde{f}(s)-\frac{\bar{\Delta}}{s}\right)\frac{e^{e_1s}}{e^{e_1s}-e^{-e_1s}}$$

$$c_3(s)=-\frac{g_1}{g_1-g_2}\left(\tilde{f}(s)-\frac{\bar{\Delta}}{s}\right)\frac{e^{-e_2s}}{e^{e_2s}-e^{-e_2s}}$$

and

$$c_4(s)=\frac{g_1}{g_1-g_2}\left(\tilde{f}(s)-\frac{\bar{\Delta}}{s}\right)\frac{e^{e_2s}}{e^{e_2s}-e^{-e_2s}} \tag{24}$$

The exponential factors appearing in equation (24) can be expanded in power series, i.e.,

$$\frac{e^{-e_ks}}{e^{e_ks}-e^{-e_ks}}=\sum_{n=0}^{\infty}e^{-2(n+1)e_ks}$$

and

$$\frac{e^{e_ks}}{e^{e_ks}-e^{-e_ks}}=\sum_{n=0}^{\infty}e^{-2ne_ks}\quad(k=1,2) \tag{25}$$

Knowing the solution for $\tilde{u}(\bar{x},s)$ and $\tilde{v}=\tilde{v}(\bar{x},s)$, the inverse transform can be obtained. The result is

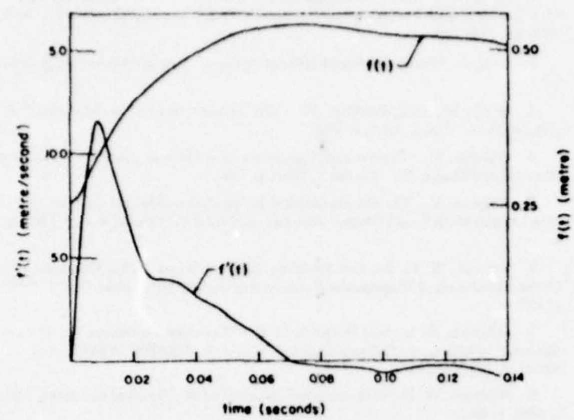

Fig. 2 Displacement and velocity curve of impacted end of spring

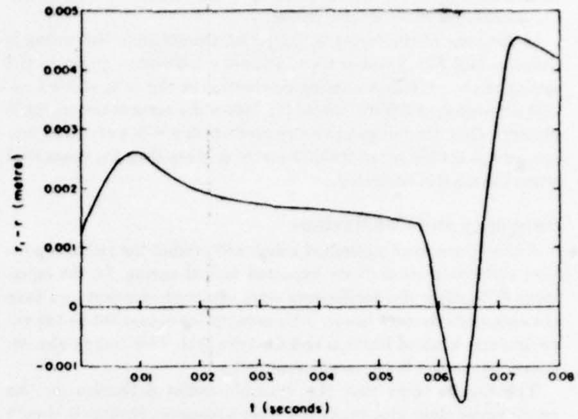

Fig. 4 Radial expansion at impacted end of spring

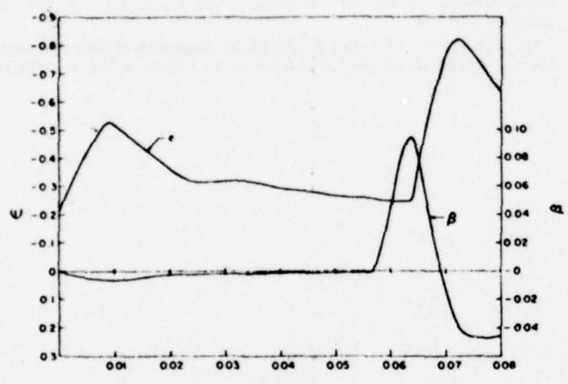

Fig. 3 Extensional and rotational strain at the impacted end of spring

$$
\begin{aligned}
\bar{u}(\bar{x},\bar{t}) = &-\frac{g_2}{g_2-g_1}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_1(2n+2-\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_1(2n+2-\bar{x})]+\frac{g_2}{g_2-g_1}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_1(2n+\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_1(2n+\bar{x})]-\frac{g_1}{g_1-g_2}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_2(2n+2-\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_2(2n+2-\bar{x})]+\frac{g_1}{g_1-g_2}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_2(2n+\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_2(2n+\bar{x})]+(1-\bar{x})\,\bar{\Delta} \qquad (26)
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{v}(\bar{x},\bar{t}) = &-\frac{g_1g_2}{g_2-g_1}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_1(2n+2-\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_1(2n+2-\bar{x})]+\frac{g_1g_2}{g_2-g_1}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_1(2n+\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_1(2n+\bar{x})]-\frac{g_1g_2}{g_1-g_2}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_2(2n+2-\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_2(2n+2-\bar{x})]+\frac{g_1g_2}{g_1-g_2}\sum_{n=0}^{\infty}\{\bar{f}[\bar{t}-e_2(2n+\bar{x})]-\bar{\Delta}\} \\
&\times H[\bar{t}-e_2(2n+\bar{x})] \qquad (27)
\end{aligned}
$$

where $H(t)$ is the unit step function.

The local strains $\epsilon = \partial u/\partial x$ and $\beta = \partial v/\partial x = r(\partial\phi/\partial x)$ can be obtained by differentiating equations (26) and (27).

Numerical Results

Numerical results are presented for a given spring with the following characteristics:

h = 1.1938 metre
r = 0.157607 metre
c = radius of spring wire = 0.0013081 metre
Δ = 0.254 metre
E = 20.68×10^{10} Newton/metre^2
M = 2.12868 kilogram
ν = 0.25
Number of coils = 6

Fig. 2 shows an experimentally obtained displacement and velocity curve for the impacted end of a spring with the properties just listed. The ends of the spring were restrained against rotation.

In order to illustrate the radial expansion of the spring, the values of the extensional strain ϵ and the rotational strain β are computed at the impacted end $x = 0$. The following equations determine ϵ and β at $x = 0$:

$$
\begin{aligned}
\epsilon(0,t) = &-\frac{g_2e_1k}{(g_2-g_1)h}\sum_{n=1}^{\infty}f'(t-2ne_1k)H[t-2ne_1k] \\
&-\frac{g_2e_1k}{(g_2-g_1)h}\sum_{n=1}^{\infty}f'[t-2(n-1)e_1kH[t-2(n-1)e_1k] \\
&-\frac{g_1e_2k}{(g_1-g_2)h}\sum_{n=1}^{\infty}f'(t-2ne_2k)H[t-2ne_2k] \\
&-\frac{g_1e_2k}{(g_1-g_2)h}\sum_{n=1}^{\infty}f'[t-2(n-1)e_2k] \\
&\times H[t-2(n-1)e_2k]-\frac{\Delta}{h} \qquad (28)
\end{aligned}
$$

and

$$
\begin{aligned}
\beta(0,t) = &-\frac{g_1g_2e_1k}{(g_2-g_1)h}\sum_{n=1}^{\infty}f'[t-2ne_1k)H[t-2ne_1k] \\
&-\frac{g_1g_2e_1k}{(g_2-g_1)h}\sum_{n=1}^{\infty}f'[t-2(n-1)e_1k]H[t-2(n-1)e_1k] \\
&-\frac{g_1g_2e_2k}{(g_1-g_2)h}\sum_{n=1}^{\infty}f'(t-2ne_2k)H[t-2ne_2k] \\
&-\frac{g_1g_2e_2k}{(g_1-g_2)h}\sum_{n=1}^{\infty}f'[t-2(n-1)e_2k]H[t-2(n-1)e_2k] \qquad (29)
\end{aligned}
$$

where

$$
k = (Mr^2h/EI)^{1/2} \qquad (30)
$$

Fig. 3 shows plots of ϵ and β, at $x = 0$, as a function of time while Fig. 4 shows the radial expansion, at $x = 0$, as a function of time as determined by equations (9) and (10). The plots in Fig. 3 and 4 are shown only for times up to 0.08 sec.

If the mass of the spring is neglected, the strain in the spring is uniform and Fig. 2 would then indicate a maximum strain in the spring of $\epsilon = -0.453$. Assuming no rotation at the ends yields a radial expansion of 0.00220 metre [1]. When the mass of the spring is not neglected, the foregoing theory predicts at $x = 0$, a radial expansion at $t = 0.1362$ sec of 0.00573 metre or more than $2\frac{1}{2}$ times that when the mass is neglected.

Summary and Conclusions

A theory has been presented which will predict the radial expansion and contraction of an impacted helical spring. In the equations of motion, the coefficients were assumed constant and thus the equations become linear. This assumption is verified by the experimental work of Phillips and Costello [11]. This theory also assumes no contact between adjacent coils.

The results show that the dynamic radial deflection can be much larger than that predicted from a massless theory. It should also be emphasized that a large portion of the deflection is due to reflections from the fixed end; a condition neglected in the work of Stokes [12].

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Reprinted from: The Journal of the Acoustical Society of America

Received 9 June 1971 12.5, 12.3, 12.3i

Large Deflections of Impacted Helical Springs

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A theoretical formulation of the large deflections of helical springs is given, and coupled nonlinear equations of motion for a typical spring element are derived. Linearized forms of these equations are solved numerically and compared with experimentally obtained streak photographs of an impacted spring. The agreement between theory and experiment is good, as long as adjacent coils of the spring do not touch.

LIST OF SYMBOLS

c radius of spring wire, or other characteristic dimension of wire cross section if wire is not circular
c_f "fast" characteristic wavespeed
c_s "slow" characteristic wavespeed
E Young's modulus of spring material
F axial force
G shear modulus of spring material
h, h_1 length of spring in unstretched and stretched positions, respectively
I moment of inertia of wire cross section
J polar moment of inertia of wire cross section
L total length of wire in spring, assumed constant
M total mass of spring
m mass of projectile impacting the spring
p, p_1 pitch of spring in unstretched and stretched positions, respectively
r, r_1 radius of spring helix in unstretched and stretched positions, respectively
S maximum shear stress in spring cross section
t time
T twisting moment about x axis
u axial displacement of spring element
v azimuthal displacement of spring element
v_0 impact velocity of projectile, positive towards spring
x axial coordinate
α, α_1 angle which the tangent to the helix makes with a plane perpendicular to the axis of the helix, in the unstretched and stretched positions, respectively
β rotational strain, $\theta r/h$ or $r\partial\phi/\partial x$
γ, γ_1 total helix angle of spring in unstretched and stretched positions, respectively
Δ change in total spring length from unstretched position, positive for extension
ϵ extensional strain, Δ/h or $\partial u/\partial x$
θ angle of twist of spring about x axis
κ, κ_1 curvature of helix in unstretched and stretched positions, respectively
ν Poisson's ratio of spring material
σ maximum bending stress in spring cross section
τ, τ_1 torsion of helix in unstretched and stretched positions, respectively
ϕ rotation of spring about x axis

INTRODUCTION

In many engineering problems involving the dynamics of springs, the mass of the spring can be neglected and it is usually assumed that the spring is linear. These assumptions allow one to formulate the problem so that in many cases a closed-form solution is possible. The problem becomes much more complex if one considers the mass of the spring and assumes that large displacements of the spring are possible.

Love[1] presents expressions for the static response of helical springs subjected to large deflections. In a more recent work, Johnson[2] lists formulas and derivations of a design procedure for dynamically loaded extension and compression springs. In an article by Dick,[3] the analysis of a simple shock wave in a helical spring is discussed. Geballe[4] reviews a theory presented by Krebs and Weidlich.[5] This treatment is correct only when the spring extensions lie in a limited region near the

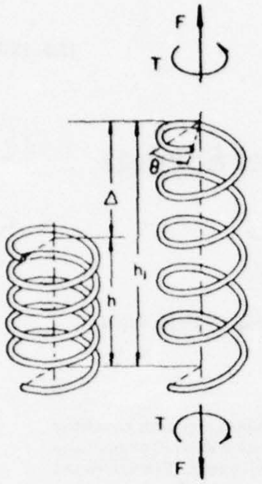

FIG. 1. Static deflection of a helical spring.

unloaded length. Durant[6] derives an expression for the stress in a dynamically loaded spring in compression when the helix angle is small. In a paper by Kagawa,[7] the longitudinal and the torsional vibrations of helical springs of finite length with small pitch are analyzed. A wide-band short-duration pulse technique was used by Britton and Langley[8] to investigate the stress wave propagation in helical springs. In a paper by Johnson and Stewart,[9] transfer functions are presented for helical springs. Wittrick[10] considers waves in springs with large helix angle.

With the exception of Love, the investigators mentioned above have restricted their analyses to *small* displacements about an equilibrium position. It is the purpose of this paper to extend Love's analysis of large static deflections to the case of *large* dynamic displacements. Of particular interest is the explicit solution $u(x,t)$ for an impacted spring.

I. THEORY

A. Force–Strain Relationships

Formally, the *static* end displacements Δ and θ of a helical spring with material properties E, ν and geometric parameters h, r, c, p can be written as

$$\Delta = f_1(F,T,E,\nu,h,r,c,p) \tag{1}$$

and

$$\theta = f_2(F,T,E,\nu,h,r,c,p), \tag{2}$$

where F and T are the axial force and torque illustrated in Fig. 1. Applying the theory of dimensional analysis to Eq. 1, one can obtain the more useful relationship

$$\frac{\Delta}{r} = f_3\left(\frac{F}{Er^2}, \frac{T}{Er^3}, \nu, \frac{c}{r}, \frac{p}{r}, \frac{h}{r}\right), \tag{3}$$

or, since Δ is a linear function of h,

$$\frac{\Delta}{h} = f_4\left(\frac{F}{Er^2}, \frac{T}{Er^3}, \nu, \frac{c}{r}, \frac{p}{r}\right). \tag{4}$$

Similar remarks can be made concerning Eq. 2 and hence

$$\frac{\theta r}{h} = f_5\left(\frac{F}{Er^2}, \frac{T}{Er^3}, \nu, \frac{c}{r}, \frac{p}{r}\right). \tag{5}$$

Now, for a given spring, ν, c/r, and p/r are constants, and therefore Eqs. 4 and 5 can be written as

$$\epsilon = \frac{\Delta}{h} = f_6\left(\frac{F}{Er^2}, \frac{T}{Er^3}\right) \tag{6}$$

and

$$\beta = \frac{\theta r}{h} = f_7\left(\frac{F}{Er^2}, \frac{T}{Er^3}\right). \tag{7}$$

These equations can be inverted to read

$$F/Er^2 = f(\epsilon,\beta) \tag{8}$$

and

$$T/Er^3 = g(\epsilon,\beta). \tag{9}$$

In order to deduce the functional form of f and g, one may now refer to Love's work (Ref. 1, p. 415) on the general theory of bending and twisting of thin rods. At this point the analysis is restricted to wire cross sections having equal principal radii of gyration, such as circular or square cross sections. Then

$$\frac{Fr^2}{EI} = \frac{r}{r_1}\frac{\cos\alpha_1}{1+\nu}\left(\frac{r}{r_1}\sin\alpha_1\cos\alpha_1 - \sin\alpha\cos\alpha\right) - \frac{r}{r_1}\sin\alpha_1\left(\frac{r}{r_1}\cos^2\alpha_1 - \cos^2\alpha\right) \tag{10}$$

and

$$\frac{Tr}{EI} = \frac{\sin\alpha_1}{1+\nu}\left(\frac{r}{r_1}\sin\alpha_1\cos\alpha_1 - \sin\alpha\cos\alpha\right) + \cos\alpha_1\left(\frac{r}{r_1}\cos^2\alpha_1 - \cos^2\alpha\right). \tag{11}$$

Making use of the geometric relations

$$p = 2\pi r\tan\alpha, \quad p_1 = 2\pi r_1\tan\alpha_1, \tag{12a,b}$$

$$h = L\sin\alpha, \quad h_1 = L\sin\alpha_1, \tag{13a,b}$$

$$\Delta = h_1 - h, \tag{14}$$

$$\gamma = h/r\tan\alpha, \quad \gamma_1 = h_1/r_1\tan\alpha_1, \tag{15a,b}$$

and

$$\theta = \gamma_1 - \gamma, \tag{16}$$

one determines that

$$\epsilon = \Delta/h = \sin\alpha_1/\sin\alpha - 1 \tag{17}$$

and

$$\beta=\frac{\theta r}{h}=\frac{1}{\sin\alpha}\left(\frac{r}{r_1}\cos\alpha_1-\cos\alpha\right). \tag{18}$$

At this stage, a slight digression yields formulas for stresses in the wire, at least for a circular cross section; this information is not required for the discussion to follow. Recalling that the curvature and torsion of a helix are given by

$$\kappa=\cos^2\alpha/r,\qquad \kappa_1=\cos^2\alpha_1/r_1, \tag{19a,b}$$

$$\tau=\sin\alpha\cos\alpha/r,\quad \tau_1=\sin\alpha_1\cos\alpha_1/r_1, \tag{20a,b}$$

and that the bending moment and torque are equal to $EI(\kappa_1-\kappa)$ and $GJ(\tau_1-\tau)$, respectively, one obtains from elementary strength of materials formulas the relations

$$\frac{\sigma}{E}=\frac{c}{r}\left(\frac{r}{r_1}\cos^2\alpha_1-\cos^2\alpha\right) \tag{21}$$

and

$$\frac{S}{G}=\frac{c}{r}\left(\frac{r}{r_1}\sin\alpha_1\cos\alpha_1-\sin\alpha\cos\alpha\right). \tag{22}$$

By a suitable combination of Eqs. 10, 11, 17, and 18, the functions f and g in Eqs. 8 and 9 are found. The results are

$$\frac{Fr^2}{EI}=(\beta\sin\alpha+\cos\alpha)\sin\alpha\left\{-\frac{\nu}{1+\nu}(1+\epsilon)(\beta\sin\alpha+\cos\alpha)+\frac{(1+\epsilon)\cos^2\alpha}{[1-(1+\epsilon)^2\sin^2\alpha]^{1/2}}-\frac{\cos\alpha}{1+\nu}\right\} \tag{23}$$

and

$$\frac{Tr}{EI}=\frac{1}{1+\nu}(1+\epsilon)\sin^2\alpha[(1+\epsilon)(\beta\sin\alpha+\cos\alpha)-\cos\alpha]+(\beta\sin\alpha+\cos\alpha)[1-(1+\epsilon)^2\sin^2\alpha]-[1-(1+\epsilon)^2\sin^2\alpha]^{1/2}\cos^2\alpha. \tag{24}$$

It should be borne in mind that Eqs. 23 and 24 have been derived by considering the total deflection of the spring of length h. It is assumed in Sec. I-B, however, that these equations can be used to compute F and T at any section of the spring in the dynamic case in terms of ϵ and β, as long as one associates with ϵ and β the *local* strains

$$\epsilon=\partial u/\partial x \tag{25}$$

and

$$\beta=r\partial\phi/\partial x. \tag{26}$$

B. Equations of Motion

Consideration of the variations of F and T with position leads to the equations of motion for a typical spring element (see Fig. 2). A summation of forces in the x direction yields

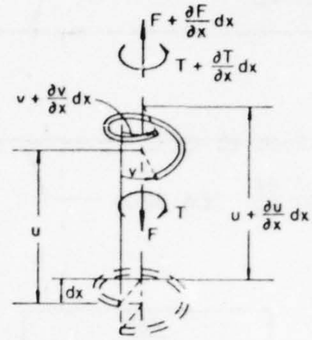

FIG. 2. Free body diagram of a spring element.

$$\frac{\partial F}{\partial\epsilon}\frac{\partial^2u}{\partial x^2}+\frac{\partial F}{\partial\beta}r\frac{\partial^2\phi}{\partial x^2}=\frac{M}{h}\frac{\partial^2u}{\partial t^2}, \tag{27}$$

while a summation of moments about the x axis yields

$$\frac{\partial T}{\partial\epsilon}\frac{\partial^2u}{\partial x^2}+\frac{\partial T}{\partial\beta}r\frac{\partial^2\phi}{\partial x^2}=\frac{Mr^2}{h}\frac{\partial^2\phi}{\partial t^2}. \tag{28}$$

In these equations, the small change in radius r has been neglected. In terms of the dimensionless variables

$$\bar{x}=x/h,\quad \bar{r}=r/h,\quad \bar{u}=u/h,\quad \bar{v}=r\phi/h,\quad \bar{t}=t/(Mhr^2/FI)^{1/2}, \tag{29}$$

Eqs. 27 and 28 can be written

$$\frac{r^2}{EI}\frac{\partial F}{\partial\epsilon}\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{r^2}{EI}\frac{\partial F}{\partial\beta}\frac{\partial^2\bar{v}}{\partial\bar{x}^2}=\frac{\partial^2\bar{u}}{\partial\bar{t}^2} \tag{30}$$

and

$$\frac{r}{EI}\frac{\partial T}{\partial\epsilon}\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{r}{EI}\frac{\partial T}{\partial\beta}\frac{\partial^2\bar{v}}{\partial\bar{x}^2}=\frac{\partial^2\bar{v}}{\partial\bar{t}^2}. \tag{31}$$

These second-order coupled differential equations are nonlinear, since the coefficients are functions of the strains ϵ and β; explicitly, one obtains from Eqs. 23 and 24 by partial differentiation

$$\frac{r^2}{EI}\frac{\partial F}{\partial\epsilon}=(\beta\sin\alpha+\cos\alpha)\sin\alpha\left\{\frac{-\nu}{1+\nu}(\beta\sin\alpha+\cos\alpha)+\frac{\cos^2\alpha}{[1-(1+\epsilon)^2\sin^2\alpha]^{3/2}}\right\}, \tag{32a}$$

$$\frac{r^2}{EI}\frac{\partial F}{\partial\beta}=\frac{r}{EI}\frac{\partial T}{\partial\epsilon}=\sin^2\alpha\left\{\frac{(1+\epsilon)\cos^2\alpha}{[1-(1+\epsilon)^2\sin^2\alpha]^{1/2}}-\frac{\cos\alpha}{1+\nu}-\frac{2\nu}{1+\nu}(1+\epsilon)(\beta\sin\alpha+\cos\alpha)\right\}, \tag{32b}$$

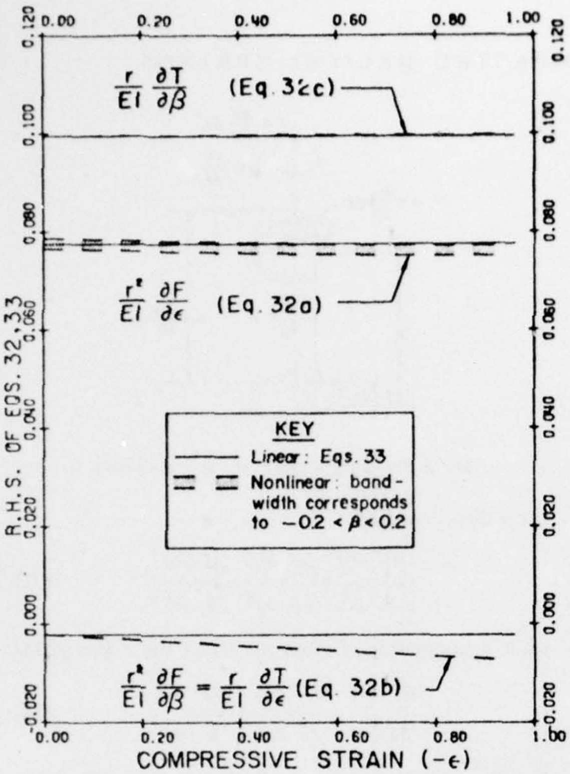

FIG. 3. Dependence of the coefficients in the wave equations on the strains ϵ and β. ($\alpha=0.10$, $\nu=0.29$.)

and

$$\frac{r}{EI}\frac{\partial T}{\partial \beta}=\sin\alpha\left[1-\frac{\nu}{1+\nu}(1+\epsilon)^2\sin^2\alpha\right]. \tag{32c}$$

It may be seen from Eqs. 32 that when the strains are small, i.e.,

$$|\epsilon|\ll 1, \quad |\beta|\ll 1, \tag{33a}$$

the coefficients have the approximate values

$$\frac{r^2}{EI}\frac{\partial F}{\partial \epsilon}\simeq\sin\alpha\left(1-\frac{\nu}{1+\nu}\cos^2\alpha\right), \tag{33b}$$

$$\frac{r^2}{EI}\frac{\partial F}{\partial \beta}=\frac{r}{EI}\frac{\partial T}{\partial \epsilon}\simeq-\frac{\nu}{1+\nu}\sin^2\alpha\cos\alpha, \tag{33c}$$

and

$$\frac{r}{EI}\frac{\partial T}{\partial \beta}\simeq\sin\alpha\left(1-\frac{\nu}{1+\nu}\sin^2\alpha\right). \tag{33d}$$

If these limiting values are employed in place of the actual nonconstant coefficients, the equations of motion, Eqs. 30 and 31, are rendered linear.

C. Initial and Boundary Conditions

The specific problem of interest is a spring which at $t=0$ is compressed uniformly, is untwisted, and is motionless:

$$u(x,0)=x\Delta/h, \quad v(x,0)=0, \quad 0\le x\le h; \tag{34a,b}$$

$$\frac{\partial u}{\partial t}(x,0)=0, \quad \frac{\partial v}{\partial t}(x,0)=0, \quad 0\le x\le h. \tag{35a,b}$$

In Eq. 34a, the displacement $u(h,0)$, namely Δ, may be regarded as being imposed by a mechanical stop.

The end of the spring at $x=0$ remains fixed for all time:

$$u(0,t)=0, \quad v(0,t)=0, \quad t>0, \tag{36a,b}$$

while the end at $x=h$ is impacted axially by a projectile of mass m, and is assumed not to rotate:

$$F(h,t)=-m\frac{\partial^2 u(h,t)}{\partial t^2}, \quad v(h,t)=0, \quad t>0. \tag{37a,b}$$

Of course, the impact velocity of the projectile must also be prescribed:

$$(\partial u/\partial t)(h,0^+)=-v_0. \tag{38}$$

It is anticipated that, at a certain instant in time, the impacted end will return to its initial position. After this instant, the projectile is no longer in contact with the spring, so that Eq. 37a no longer applies; rather, owing to the presence of the mechanical stop, either

$$u(h,t)=\Delta \quad \text{or} \quad F(h,t)=0, \tag{39a,b}$$

the latter condition holding only if and when dynamic separation occurs.

II. NUMERICAL RESULTS

A. Examination of Coefficients

It is interesting to note that for springs of moderate helix angle (say $\alpha=0.1$), the coefficients $\partial F/\partial\epsilon$, etc., appearing in the equations of motion, Eqs. 30 and 31, are rather insensitive to the magnitude of the strains ϵ and β, at least in the range of compressive axial strain ($-1<\epsilon<0$). Specifically, in Fig. 3 one sees that with the exception of the small "cross terms" $\partial T/\partial\epsilon$ and $\partial F/\partial\beta$, these coefficients do not vary more than a few percent from their corresponding values at zero strain, given by Eq. 33. (Note that only $\partial F/\partial\epsilon$ shows any significant dependence on the rotational strain β.) Nevertheless, it was anticipated that use of the general nonconstant coefficients (Eq. 32) would be required to describe the spring response accurately, and for this reason a generally applicable numerical solution technique was sought.

B. Finite-Difference Solution

Of the methods available, the method of finite differences[11] seemed the simplest to use. Spatial and temporal second-order central difference formulas were used for all the second partial derivatives appearing in Eqs. 30, 31, and 37. The first-order difference

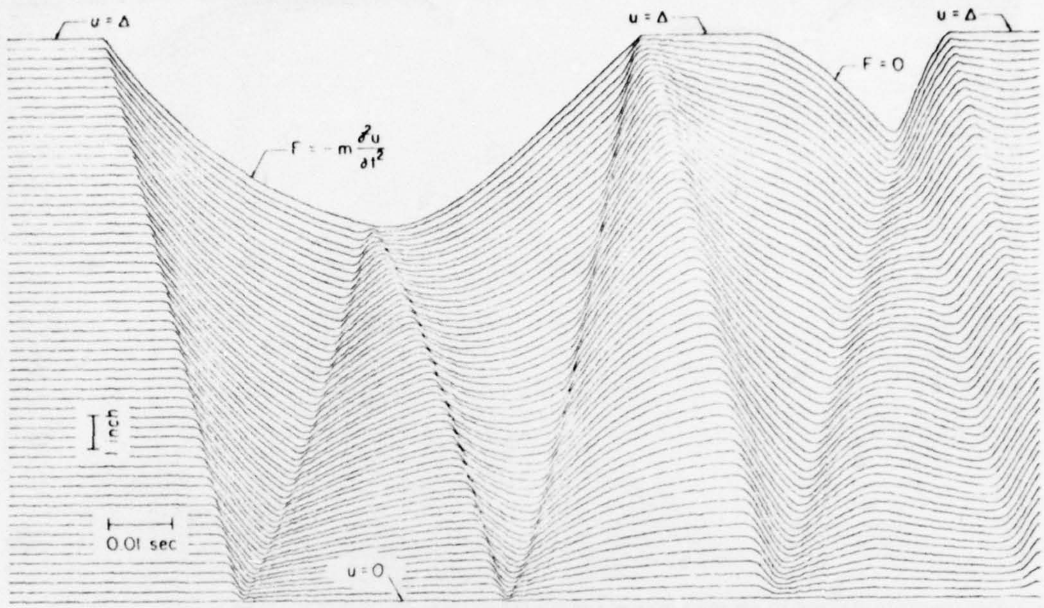

FIG. 4. Theoretical axial positions, as functions of time, of all the coils of a 58-coil spring lightly precompressed ($\Delta/h = -0.03$) and impacted with a projectile. Results are from the linear theory. Parameters are given in Eq. 40.

relations for the initial conditions, Eqs. 35 and 38, were integrated explicitly over the first time step, after which the general solution was allowed to march forward in time.

The following spring parameters were used:

$$\begin{aligned} h&=18.3 \text{ in.},\\ c&=0.050 \text{ in. (circular wire)},\\ r&=0.50 \text{ in.},\\ p&=h/58,\\ E&=30\times10^6 \text{ lb/in.}^2,\\ \nu&=0.29,\\ M&=1.07\times10^{-3} \text{ lb sec}^2\text{/in.},\\ m&=1.67\times10^{-3} \text{ lb-sec}^2\text{/in.},\\ v_0&=275 \text{ in./sec.} \end{aligned} \tag{40}$$

These values correspond to data for a 58 coil steel spring used in the experimental program discussed later. For this spring, the helix angle α is 0.10, so that the information in Fig. 3 is pertinent.

C. Results

The axial position $x_i + u(x_i,t)$ of the ith coil has been plotted in Fig. 4 for all the coils ($i=0,1,2,3,\cdots,58$) as a *function of time* t, for an impacted spring which has been lightly precompressed. It must be recalled that only over that portion of the top curve labeled "$F = -m\partial^2 u/\partial t^2$" is the projectile actually in contact with the end coil. Otherwise the mechanical stop limits the maximum end coil displacement. The results in Fig. 4 are based upon the linear theory, i.e., upon use of the *constant* coefficients Eqs. 33.

The impacting mass sets up an initial wavefront which is straight, because linear theory is being used and the initial spring configuration is one of uniform strain. All reflected wavefronts pass through regions of nonuniform strain, however, and consequently they appear curved even though in the *undeformed* coordinate x the wavefronts are straight.

Actually, two distinct wavefront slopes in (x,t) space are possible in this problem. Specifically, by considering the characteristic surfaces[12] associated with the governing equations, Eqs. 30 and 31, one can show that discontinuities in the variables F, T, $\partial u/\partial t$, $\partial\phi/\partial t$ can be propagated at the speeds c_f and c_s given by

$$c_f^2=\frac{Elh}{Mr^2}\sin\alpha,\quad c_s^2=\frac{Elh}{Mr^2}\frac{\sin\alpha}{1+\nu} \tag{41a,b}$$

in the undeformed coordinate x when the linear theory is used. For moderate $\alpha(<0.1$, say), jumps in the axial velocity $\partial u/\partial t$ are propagated principally along the slower wavefront, whereas jumps in the rotational velocity $\partial\phi/\partial t$ are propagated principally along the faster wavefront. Since it is the axial displacement u which is represented in Fig. 4, the wavefronts in that figure should correspond to the slow wavespeed c_s, and this is in fact verified by a comparison of the values of c_f and c_s obtained from Eqs. 40 and 41 with the slope which the initial wavefront in Fig. 4 would have in (x,t) space.

After the projectile leaves the spring, the end coil remains at the mechanical stop briefly and then separates, only to return shortly thereafter. This dynamic separation, which is enhanced by large impact

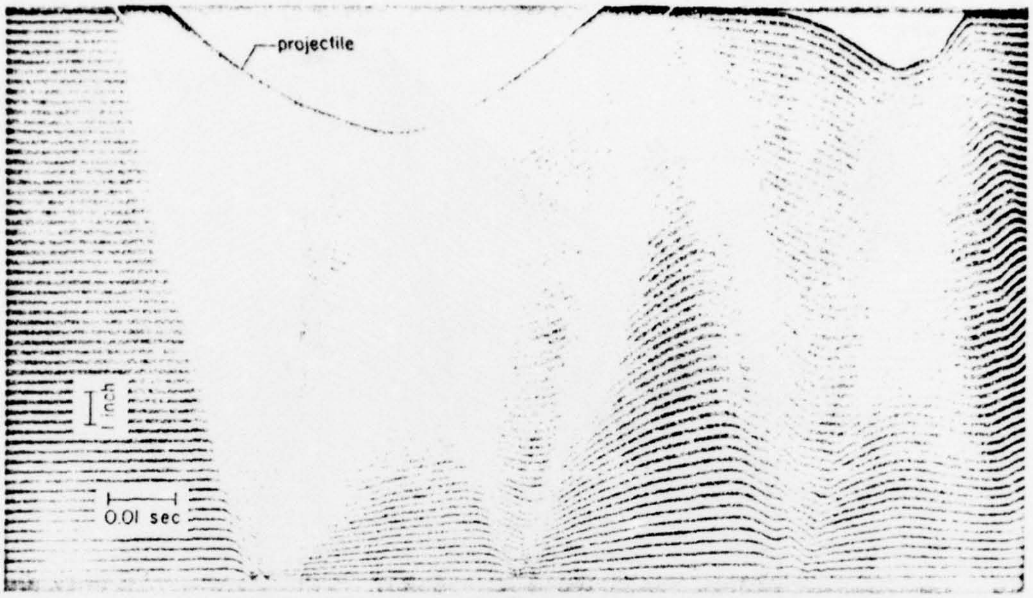

FIG. 5. Experimentally obtained streak photograph of a spring lightly precompressed ($\Delta/h=-0.03$) and impacted with a projectile. Parameters are given in Eq. 40.

velocities and light precompressions, occurs again and again with some regularity for times greater than that shown in Fig. 4.

Remarks about use of the nonlinear theory are made later in the paper; no results based on the nonlinear theory will be presented. However, it should be repeated that the finite-difference technique outlined above can handle the nonlinear case as well as the linear one.

III. EXPERIMENT

It is interesting to compare the results of the linear theory to actual streak photographs of impacted springs, such as that shown in Fig. 5. for a spring and projectile having the properties, Eq. 40.

A. Experimental Setup

Several records like those shown in Figs. 5 and 6 were obtained with the aid of a streak camera of the rotating drum type. In the experimental setup, a horizontal rod supported the spring specimen and the thick-walled cylindrical projectile axially. One end of the spring was rigidly fixed, while the end facing the projectile was brought to its precompressed position by means of a thin washer and a retaining plate through which the projectile could pass freely. The optical axis of the camera was horizontal and perpendicular to the spring axis; at any instant in time, only a series of dots corresponding to the instantaneous positions of all the coils appeared on the photographic paper wrapped around the drum. To trigger the shutter on the camera, suitable delay circuits were constructed and synchronized with a photocell monitoring the path of the projectile just prior to impact.

B. Results

In Figs. 5 and 6, just as in Fig. 4, the bottom line represents the fixed end of the spring as a function of time, while the top line represents the position of the impacted end. The series of parallel lines on the left-hand side correspond to the undisturbed initial positions of all the coils in the precompressed configuration, and the black streak labeled "projectile" corresponds to the trace of the 2-in.-long projectile as a function of time.

If one's attention is confined to Fig. 5 alone, it is observed that all the features predicted by the linear theory are found experimentally: a straight initial wavefront, seemingly curved reflected wavefronts, approximately two interactions of reflected wavefronts with the projectile, and dynamic separation shortly after the projectile leaves. Furthermore, the linear theory predicts accurately the wavespeed associated with jumps in the axial velocity, the maximum penetration of the projectile, and the time of projectile contact.

In Fig. 6, however, where the magnitudes of the precompression and impact velocity are considerably greater than those in Fig. 5, there is evidence of contact between adjacent coils. This contact phenomenon, which was not considered in the development of the theory, accounts for the curved *initial* wavefront and the near-vertical reflected wavefront at the fixed end.

IV. CONCLUSIONS

For moderate helix angle α, it has been shown that the linear theory developed in this paper is adequate for describing the dynamic spring response as long as

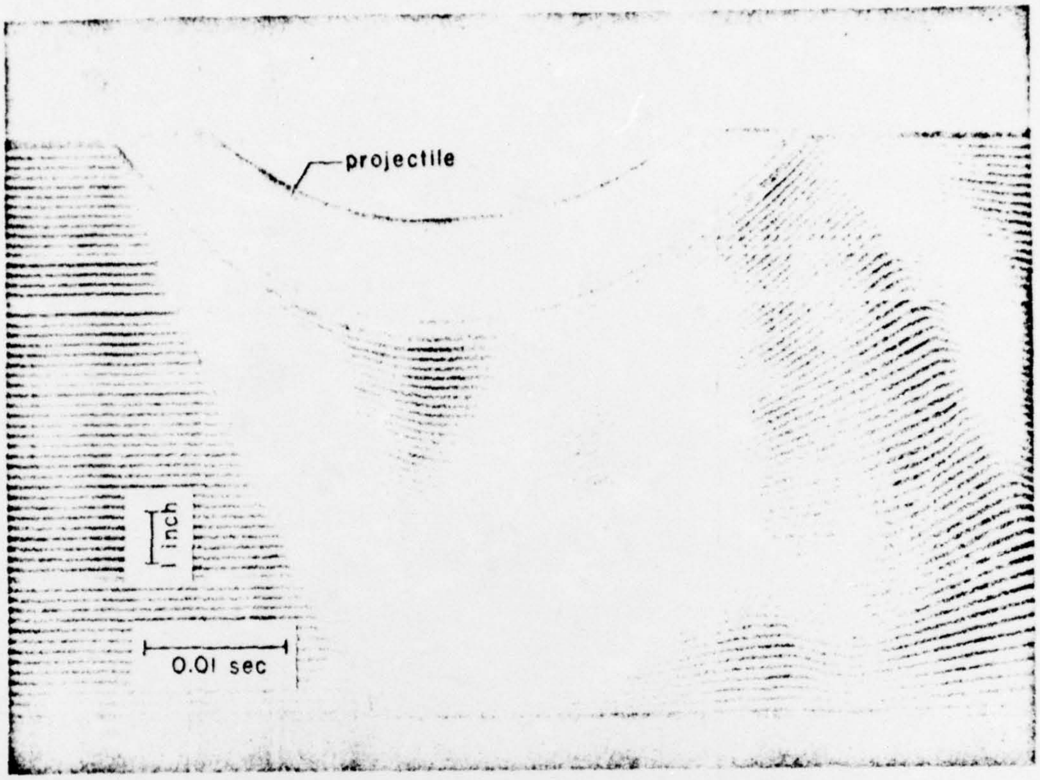

FIG. 6. Streak photograph of the same spring and projectile as in Fig. 5, but with a larger precompression ($\Delta/h = -0.39$) and larger impact velocity ($v_0 = 380$ in./sec.).

the coils do not touch one another. Specifically, the good agreement between theory and experiment (Figs. 4 and 5, respectively) indicates that one may use the linear theory even when the displacements are large and the axial strains vary between zero and minus one-half, approximately.

One would expect, however, that for large helix angles and/or large positive axial strains, the linear theory may no longer hold adequately, and in this case Eqs. 30 and 31 would have to be solved in conjunction with the nonlinear force-displacement relations, Eqs. 32.

ACKNOWLEDGMENTS

The authors wish to thank Professor R. E. Miller for many valuable discussions and also Dr. Alexander Hammer, Mr. Ed Larrison, and Mr. Henry Swieskowski of the U. S. Army Weapons Command at Rock Island Arsenal, Illinois, for their support.

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Technical Report No. R-TR-76-045, Dec 1976

38 pages, Incl Figures & Tables

An analysis was conducted during the period of November 1975 to May 1976 to determine the dynamic response of helical springs under axial impact with particular application to the return spring in the M60A2 tank recoil mechanism. The study was performed by the University of Illinois at Urbana, Illinois under the direction of the General Thomas J. Rodman Laboratory, Rock Island Arsenal. The dynamic behavior of the impacted spring is described by two coupled nonlinear partial differential equations. Two computer solutions to the equations are obtained by the methods of finite differences and nonlinear characteristics. Results of the two numerical techniques are

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